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PG4S-026-B-23

M.Sc. IV Semester Degree Examination

MATHEMATICS

Differential Geometry (New)

Paper : HCT 4.4

Time : 3 Hours **Maximum Marks :80**

Instruction to Candidate:

1) Solve any **FIVE** questions
2) All questions carry **Equal** Marks

1. a) Define the directional derivative. Further, let ϑ_p be the tangent vector to E^3 for which $\vartheta = (2,-1,3)$ and P=(2,0,-1). Compute the directional derivative $\vartheta_p[f]$, where i) $f = y^2 z$ ii) $f = e^x \cos y$ iii) $f = x^7$ **(08)**

b) Define reparametrization of a curve. Find a straight line passing through the point (1,-3,-1) and (6,2,1). Does this line meet the line passing through points(-1, 1, 0) and (-5, -1, -1) **(08)**

2. a) For any three 1-forms $\phi_i = \sum f_{ij} dx_j (1 \le i < 3)$ then prove that

$$\phi_1 \wedge \phi_2 \wedge \phi_3 = \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} dx_1 dx_2 dx_3$$ **(08)**

b) Define Wedge product. Further , simplify the following forms:

i) $d[fdg + gdf]$

ii) $[(f-g)(df+dg)]$

iii) $d[fdg \wedge gdf]$

iv) $d(gfdf) + d(fdg)$ **(08)**

3. a) Establish the Frenet formulae for an unit speed curve $\beta : I \to E^3$ with $K > 0$ **(08)**

b) if β be a unit speed curve in E^3, with curvature K>0, then prove that β is a plane curve if and only if $\tau = 0$ **(08)**

4. a) Define a Cylindrical helix. Further, prove that a regular curve α with $\kappa > 0$ is a cylindrical helix. if and only if $\frac{\tau}{\kappa}$ is constant **(08)**

b) Compute Frenet apparatus κ, τ, T, N, B of the unit speed curve

$$\beta(s) = \left(\frac{4}{5}\cos s, 1 - \sin s, \frac{-3}{5}\cos s\right)$$ **(08)**

5. a) Define an isometry of E^3 with $F(0) = 0$ then prove that F is an orthogonal transformation **(08)**

b) If F is an isometry of E^3 then prove that there exists a unique translation T and a unique orthogonal transformation C such that F = TC **(08)**

6. a) Define Sign of an isometry. Further, show that all translations and all rotations are orientation preserving **(06)**

b) Define Congruence of a curve. If $\alpha, \beta : T \to E^3$ are unit speed curves such that $\kappa_\alpha = \kappa_\beta$ & $\tau_\alpha = \pm \tau_\beta$ then prove that α *and* β are congruence curves **(10)**

7. a) Prove that every sphere in E^3. Is a surface in E^3 **(08)**

b) Define a Monge patch in E^3. If f be a real valued differentiable function on a non empty open set D of E^3 then show that the function $X : D \to E^3$ is defined by $X(u,v) = (u, v, f(u,v))$ is a Monge patch **(08)**

8. a) If ϕ be a 1-form on M and if X and Y are patches in M then prove that $d_x\phi = d_y\phi$ on the overlap of x(D)=y(E) **(08)**

b) Prove that a mapping $X : D \to E^3$ is a regular iff $X_u(d)$ *and* $X_\vartheta(d)$ are the u, ϑ partial derivatives of $X(u,v) = X(d)$ are linearly independent $\forall d \in D$ where D C E^3 **(08)**

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$\forall d \in D$ where D C E^3 **(08)**

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M.Sc. IV Semester Degree Examination
MATHEMATICS
Computational Numerical Methods - II
Paper : HCT 4.3

Time : 3 Hours **Maximum Marks :80**

Instructions to Candidate:

1) Answer any **five** full questions.
2) **All** questions carry equal marks.

1. a) Describe Eulers method for solving initial value problem $y' = f(x, y)$ with $y(x_0)=y_0$. **(8)**

b) Use Runge-Kutta fourth order method to approximate y at x=0.2, given that y(0)=1 and $y' = x+y^2$ (use the step size h=0.1) **(8)**

2. a) Describe Runge-Kutta method for solving simultaneous first order differential equations and hence solve

$y'' = xy' - y; y(0) = 3, y'(0) = 0$ and find $y(0.1)$. **(8)**

b) Use Milne's method to find $y(0.3)$ from $y' = x^2 + y^2$; $y(0) = 1$. Find the initial values $y(-0.1)$, $y(0.1)$ and $y(0.2)$ from the Taylors series method. **(8)**

3. a) Describe Adam's-Bashforth predictor corrector method of solving an initial value problem $y' = f(x, y)$ with $y(x_0)=y_0$. **(8)**

b) Write the classification of general second order partial differential equation and discuss the classificaion of physical problems. **(8)**

4. a) Derive explicit finite difference scheme to solve general parabolic partial differential equation. **(8)**

b) Use implicit finite difference Scheme to solve the IBVP $u_t=u_{xx}+(x-2)u_x-3u$ with initial condition $u(x,0) = x^2 - 4x + 5, 0 \le x \le 4, t > 0$; and the boundary conditions $u(0,t)= u(4,t)=5e^{-t}, t>0$. **(8)**

5. a) Use crank-Nicholson method to solve $u_t = u_{xx}$ with $u(x,0) = \sin \pi x;\ 0 \le x \le 1; u(0,t) = u(1,t) = 0$, for one time step. (8)

b) Use ADI method to solve upto 2 levels of time of $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ with the initial condition $u(x,y,0) = \sin \pi x \sin \pi y, \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}$ and boundary conditions

$$\left.\begin{matrix} u(x,y,t) = 0 \ for\ x = 0\ and\ x = 1 \\ u(x,y,t) = 1\ for\ y = 0\ and\ y = 1 \end{matrix}\right\} for\ t > 0$$

Take $h = \frac{1}{3}$, K=1 so that $A = \frac{1}{9}$. (8)

6. a) Derive standard five-point formula and diagonal five - point formula to solve the Laplace equation $u_{xx}+u_{yy}=0$. (8)

b) Solve the Laplace equation $u_{xx}+u_{yy}=0$ with the boundary conditions $u(0,y)=0$, $u(x,0)=0$ $u(1,y)=100y$, $u(x,1)=100x$ by dividing the region into square mesh with a grid spacing of $h=0.25$. (8)

7. Solve $u_{xx} + u_{yy} = 9u : \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}$ with u satisfying the above equation at every point inside the square and is subject to the bounadry conditions

$u=x$ at $y=0$
$u=x+1$ at $y=1$

$$\left.\begin{matrix} \frac{\partial u}{\partial x} = 2u - y \quad at\ x = 0 \\ u = 2 \quad\quad at\ x = 1 \end{matrix}\right\} 0 < y < 1 \text{ take } h = \frac{1}{3}.$$ (16)

8. a) Describe the implicit finite difference method to solve hyperbolic partial differential equation. (8)

b) Write a note on Galerkin method. (8)

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M.Sc. IV Semester Degree Examination
MATHEMATICS
Measure Theory
Paper : HCT 4.1

Time : 3 Hours **Maximum Marks :80**

Instruction to Candidate:

1) Answer any five full questions.
2) All questions carry equal marks.

1. a) Define an outer measure. Let x be a space of atleast two points and $x_0 \in X$. For each $A \subset X$, defined $\mu(A) = \begin{cases} 0 \text{ if } x_0 \notin A \\ 1 \text{ if } x_0 \in A \end{cases}$

Then prove that μ is an outer measure. (8)

b) Prove that the union and intersection of two outer measurable sets is measurable.(8)

2. a) Show that an open set in a metric space is measurable with respect to any outer measure. (8)

b) Define exterior and interior measure of a set. Show that $m_e(A) \geq m_i(A)$ for any Set A. (8)

3. a) If E_1 and E_2 are measurable then prove that $E_1 \cup E_2$ is measurable. What do you say about $E_1 \cap E_2$ is measurable? Justify your answer. (8)

b) State and prove the second fundamental theorem. (8)

4. a) If f is a measurable function defined over a measurable set E. Let 'C' be any real number then show that cf, f+c, $|f|$, f^2 are measurable functions. (8)

b) Show that a continuous function defined in a closed interval is measurable. (8)

5. a) Let $<f_n>$ be a sequence of functions which converges in measure to the function f on a measurable set E. Then prove that there exists a subsequence which also converges to the function f almost everywhere. (8)

b) Define convergence in mean. If $< fn >$ be a sequence of integrable functions which converge in mean to a function f_1 then show that $f_n \to f$ in measure also. (8)

6. a) If a function f is absolutely continuous in an interval and $f^1(x)=0$ a.e. then prove that 'f' is a constant function. **(8)**

b) Define a function of bounded variation. If $f(x)$ is continuous and integrable function and $F(x)=\int_a^x f(t)dt+F(a)$ then prove that $F^1(x)=f(x)$ a.e. **(8)**

7. a) Define an absolutely continuous function. If $f(x)$ and g(x) are obsolutely continuous functions then prove that their sum, difference and product are also obsolutely continuous function. **(8)**

b) Define an indefinite integral of a function. Prove that an indefinite integral is an absolutely continuous function. **(8)**

8. a) Define a signed mesure. If ν is a signed measure on a measurable space (X,A). Then prove that there exists a positive set P and a negative set Q such that $P\cap Q=\phi,\ X=P\cup Q$ where A being $\sigma-$ algebra of subsets of X. **(8)**

b) Let (x, A, μ) be a $\sigma-$ finite measure space and ν be a $\sigma-$ finite measure defined on A. Then prove that there exists two uniquely determined measures r_0 and r_1 such that $r=r_0+r_1$ and $r_0\perp\mu,\ r_1<<\mu$. **(8)**

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M.Sc. IV Semester Degree Examination

MATHEMATICS

Fluid Mechanics - II

Paper : SCT 4.1

Time : 3 Hours **Maximum Marks :80**

Instruction to Candidates:

1) Answer any five questions.
2) All questions carry equal marks.

1. a) Derive the relation between stresses and rate of deformation. **(8)**

b) Derive Navier-stokes equation for a viscous fluid. **(8)**

2. Derive the energy equation. **(16)**

3. a) Prove that any problem can be expressed in the form of the relationship between a group of variables i.e, $f(v_1, v_2, \ldots\ldots v_i, \ldots . v_n)=0$ where f denotes a function of some variables v_i. **(8)**

b) Explain three different types of similarities. **(8)**

4. a) Define Froude number, Eulers number, Mach number and Reynolds number. **(8)**

b) Show that by means of dimensional analysis that the thrust T of a propeller is given by $T=\rho d^2 v^2 \phi\left(\frac{vd\rho}{\mu}, \frac{dn}{v}\right)$ where ρ is the fluid density, μ its viscosity, d is the diameter of the propeller, v is the speed of advance and n is the revolution per second. **(8)**

5. a) Derive the drag coefficient for plane foiseuille flow. **(8)**

b) Explain the temperature distribution for the plane couette flow. **(8)**

6. a) Derive the flux $Q=\frac{\pi G}{4\mu} \frac{a^3 b^3}{a^2+b^2}$ for the fluid over the area of the elliptic cross-section. **(8)**

b) Derive the flux for the steady flow in pipes of rectangular section in the form,

$$Q=\frac{4c}{3\mu} ab^3-\frac{c}{\mu} \frac{256 b^4}{\pi^5} \Sigma \frac{1}{(2n+1)^5} \tanh \{(2n+1)(\pi a/2b)\}.$$ **(8)**