Roll No. ____________________ [Total No. of Pages : 2

PG3S-409-A-23

M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Functional Analysis

Paper - HCT 3.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidate:

1) Solve any **FIVE** questions.
2) All questions carry **equal** marks.

(5×16=80)

1. a) Define a metric space. Further, state and prove cantor's intersection theorem. **(8)**

   b) If (X,d) is complete metric space and Y a subspace of X. Then prove that Y is complete if and only if Y is closed. **(8)**

2. a) State and prove Bair's category theorem. **(8)**

   b) Prove that a metric space is sequentially compact if and only if it has the Bolzano Weierstrass property. **(8)**

3. a) Show that the linear space R^n and C^n of all n - tuples $x=(x_1,x_2,...,x_n)$ of real and complex numbers are Banach spaces under the norm $\|x\|=\left(\sum|xi|^2\right)^{1/2}$. **(8)**

   b) State and prove Riesz's lemma. **(8)**

4. a) State and prove Hahn-Banach theorem. **(10)**

   b) If x and y are any two vectors in a Hilbert space then show that $|(x, y)| \le \|x\|\|y\|$ **(06)**

5. a) State and prove natural embedding theorem of N into N** **(8)**

   b) If B and B' are Banach spaces and if T is continuous linear transformation of B onto B', then show that T is an open mapping. **(8)**

6. a) Let X and Y be Banach spaces and let $T: X \to Y$ be a linear transformation of X into Y. Then prove that T is continuous if its graph T_G is closed. **(8)**

   b) Define a Hilbert space. Further, state and prove Cauchy. Schwartz's inequality. **(8)**

7. a) Prove that a Banach space is a Hilbert space if and only if the parallelogram law holds. **(8)**

b) State and prove Riesz representation theorem. **(8)**

8. a) Let H be a Hilbert space and Let f be an arbitrary functional in H*, then prove that there exists a unique vector 'y' in H such that

$$f(x) = (x, y), \forall x \in H$$ **(10)**

b) Define a self adjoint operator on a Hilbert space H. Further, if T_1 and T_2 are self adjoint operators, then show that T_1T_2 is self adjoint if and only if $T_1T_2 = T_2T_1$ **(06)**

Roll No. ______________________ [Total No. of Pages : 2

PG3S-412-A-23

M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper : SCT 3.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any **Five** questions.*
2. *All questions carry **equal** marks.*

1. a) Derive the required condition for a surface to be a possible form of boundary surface. **(8)**

b) Define boundary conditions on velocity, pressure and temperature. **(8)**

2. a) Derive equation of continuity by Eulers method. **(8)**

b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t - 1 = 0$ is a possible form of boundary surface and find an expression for normal velocity. **(8)**

3. a) State and prove Helmholtz vorticity equation. **(8)**

b) Show that the difference of the values of ψ at the two points represents the flux of a fluid across any curve joining the two points. **(8)**

4. a) Define source, sink and find the complex potential for a doublet. **(8)**

b) State and prove Milne - Thomson circle theorem. **(8)**

5. a) Discuss the motion in the case of a liquid streaming past a fixed circular cylinder of radius '*a*' with velocity W. **(8)**

b) Find the pressure at points on the cylinder for $W = W\left(z + \frac{a^2}{z}\right) + \frac{iK}{2\pi}\log z$ **(8)**

6. a) A circular cylinder is placed in a uniform stream. Find the forces acting on the cylinder. **(8)**

b) A circular cylinder of radius '*a*' and infinite length lies on a plane in an infinite depth of liquid. The velocity of liquid at a great distance from the cylinder is 'W' perpendicular to generations and the motion is irrotational and two dimensional. Verify that the stream function is the imaginary part of $w = \pi\alpha W\, cot-h\left(\frac{\pi a}{z}\right) where\ z = 0$ on the line of contact. Prove that the pressure at the two ends of the diameter of the cylinder normal to the plane differs by $\frac{1}{32}\pi\, 4\,\delta\, W^2$. **(8)**

7. a) State and prove kinetic energy of infinite liquid. **(8)**

b) Show that the mean value of the velocity potential over a spherical surface using in liquid is equal to its value at the centre of the sphere. **(8)**

8. a) Show that in the motion of fluid in two dimensions, if the co - ordinates (x,y) of an element at any time can be expressed in terms of initial co - ordinates (a,b) and the time, the motion is irrotational if $\frac{\partial(\dot{x},x)}{\partial(a,b)} + \frac{\partial(\dot{y},y)}{\partial(a,b)} = 0$. **(8)**

b) Liquid of density δ is flowing in two dimensions between the oral curves $r_1r_2 = a^2$, $r_1r_2 = b^2$, where r_1, r_2 are the distances measured from two fixed points, if the motion is irrotational and quantity 'q' per unit time across any line joining the bounding curves, then the kinetic energy is $\frac{\pi\delta q^2}{wg(b/a)}$. **(8)**

Roll No. ____________________ [Total No. of Pages : 2

PG3S-413-A-23
M.Sc. III Semester Degree Examination
MATHEMATICS
Operations Research - II
Paper : OET 3.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **FIVE** questions.*
2) *All questions carry **equal** marks.*

1. a) Use Big-M method to solve the following L.P.P. **(8)**

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

and $x_1, x_2, x_3 \geq 0$

b) Use two - phase simplex method to solve the following L.P.P? **(8)**

Maximize $Z = x_1 + 2x_2 + 3x_3$

Subject to $x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_1 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$

2. a) Prove that the dual of the dual is primal. **(8)**

b) Use dual simplex method to solve the following L.P.P. **(8)**

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

Subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$

3. a) Explain : **(8)**

   i) Two - Person zero-sum games

   ii) The maximin - minimax principle.

   b) Determine the range of the value of p and q that will make the payoff element Q_{22}, a saddle point for the game whose payoff matrix (a_{ij}) is given below: **(8)**

Player B

$$\text{Player A} \begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix}.$$

4. a) Solve the following game graphically : **(8)**

Player B

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix} \end{array}$$

   b) Using the principle of dominance, solve the following game. **(8)**

Player B

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \end{bmatrix} \end{array}$$

5. a) Define a queuing system and explain in brief the characteristics of the queuing system. **(8)**

   b) Explain Pure birth process. **(8)**

6. a) Discuss $(M/M/1):(\infty/FIFO)$ model. **(8)**

   b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter - arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate :

   i) The mean queue size

   ii) The probability that the queue size exceeds 10.

   iii) If the input of trains increases to an average 32 per day, what will be the change in i) and ii). **(8)**

7. a) Write steps in simulation. **(8)**

   b) Explain the concept of event type simulation. **(8)**

8. a) Discuss the advantages and limitations of simulation models. **(8)**

   b) Discuss simulation languages. **(8)**

Roll No. ______________________ [Total No. of Pages : 2

PG3S-410-A-23

M.Sc. III Semester Degree Examination

MATHEMATICS

Graph Theory - I

Paper : HCT 3.2

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **Five** questions.*
2) *All questions carry **equal** marks.*

1. a) Define spanning and Induced subgraph of a graph. Prove that a graph is bipartite if and only if its cycles are even. **(8)**

b) Prove that every self-complementary graph has 4n or 4n+1 vertices. **(4)**

c) Define product and composition operations. Draw the following graphs:

i) $K_{1,3} \times K_{1,3}$

ii) $P_2[P_3]$ **(4)**

2. a) Write STEPS for detection of planarity of a graph. Show that K_5 and $K_{3,3}$ are nonplanar. **(8)**

b) Define a Peterson graph. Show that the Peterson graph is non-planar. **(8)**

3. a) Prove that a graph is outerplanar if and only if it has no subgraph homeomorphic to K_4 or $K_{2,3}$. **(8)**

b) Define the crossing number. Determine $Cr(K_{2,2,3}) = 2$ **(8)**

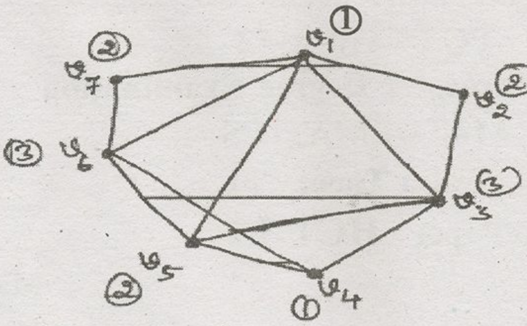
4. a) Show that every tree with two or more vertices is 2-chromatic. **(5)**

b) Find the chromatic number of the following graphs:

i) $\overline{K}p$ ii) P_5 iii) C_{16} iv) $K_{m,n}$ **(4)**

c) For any graph G, the sum of x and $\overline{x}$ satisfy the following inequality, $2\sqrt{p} \le \chi + \overline{\chi} \le p+1$. **(7)**

5. a) Illustrate the simple sequential coloring Algorithm for the graph given below: **(10)**



b) Discuss the smallest last sequential algorithm through a graph. **(6)**

6. a) Define an uniquely colorable graph. Show that every uniquely 4 - colorable planar graph is maximal planar. **(8)**

b) For any non-trivial connected graph G, Prove that $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$ *where* α_0, α_1 are the vertex and edge covering number and β_0, β_1 are the vertex and edge independence number respectively. **(8)**

7. a) Prove that a complete graph K_{2n} is 1-factorable. **(8)**

b) For any positive integer n, prove that the graph K_{2n+1}, can be factored into n-Hamiltonion cycles. Draw the factorization of K_7 into Hamiltonian cycles. **(8)**

8. a) Define the following graphs:

i) Simple digraph

ii) asymmetric digraph

iii) Symmetric digraph

iv) Complete digraph. **(4)**

b) Prove that a digraph G is an Euler digraph if and only if G is connected and is balanced. **(6)**

c) Prove that every tournament has a Hamiltonian path. **(6)**
