

087821

Roll No _____

[Total No. of Pages : 2

PG2S-284-B-23
M.Sc. II Semester Degree Examination
MATHEMATICS
Partial Differential Equations
Paper - HCT 2.1

Time : 3 Hours

Maximum Marks :80

Instructions to Candidate:

- 1) Solve any FIVE questions.
- 2) All questions Carry equal marks

1. a) Define the following for a partial differential equations with examples:
 - i) Linear Equation
 - ii) Semilinear Equation
 - iii) Quasilinear Equation
 - iv) Nonlinear Equation (8)
- b) Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ which passes through the circle $z = 1, x^2 + y^2 = 1$ (8)
2. a) Write a short note on surfaces orthogonal to a given system of surfaces. (8)
- b) Find the surface which is orthogonal to the system $z(x + y) = c(3z + 1)$ and passes through the circle $x^2 + y^2 = 1, z = 1$. (8)
3. a) Define compatible systems of first order equations, further, show that first order partial differential equations $p = p(x, y)$ and $q = Q(x, y)$ are compatible if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. (8)
- b) Show that the partial differential equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and hence find their solution. (8)

087821

Roll No _____

[Total No. of Pages : 2

PG2S-284-B-23
M.Sc. II Semester Degree Examination
MATHEMATICS
Partial Differential Equations
Paper - HCT 2.1

Time : 3 Hours

Maximum Marks :80

Instructions to Candidate:

- 1) Solve any FIVE questions.
- 2) All questions Carry equal marks

1. a) Define the following for a partial differential equations with examples:
 - i) Linear Equation
 - ii) Semilinear Equation
 - iii) Quasilinear Equation
 - iv) Nonlinear Equation (8)
- b) Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ which passes through the circle $z = 1, x^2 + y^2 = 1$ (8)
2. a) Write a short note on surfaces orthogonal to a given system of surfaces. (8)
- b) Find the surface which is orthogonal to the system $z(x + y) = c(3z + 1)$ and passes through the circle $x^2 + y^2 = 1, z = 1$. (8)
3. a) Define compatible systems of first order equations, further, show that first order partial differential equations $p = p(x, y)$ and $q = Q(x, y)$ are compatible if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. (8)
- b) Show that the partial differential equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and hence find their solution. (8)

4. Solve the following partial differential equations.

a) $t - xq = x^2$ (4)

b) $p + r + s = 1$ (8)

c) $s - t = \frac{x}{y^2}$ (4)

5. a) Solve the heat equation $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$ using variable separable method. (8)

b) The faces $x = 0$, $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f(x)$, ($0 < x < a$). Determine the temperature at a subsequent time t . (8)

6. a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it. (8)

b) A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity in the undisturbed state is V_0 . Determine the velocity of the fluid at any point of the disturbed stream. (8)

7. a) Derive the Monge's method for solving nonlinear partial differential equation of second order $Rr + Ss + Tt = V$, where R, S, T and V are functions of x, y, z, p & q . (8)

b) Solve partial differential equation $r - a^2 t = 0$ by Monge's method. (8)

8. Solve the following non-linear partial differential equations.

a) $r + 4s + t + rt - s^2 = 2$. (8)

b) $q^2 r - 2pqs + p^2 t = 0$. (8)

Roll No _____

1089143

[Total No. of Pages : 2

PG2S-286-B-23
M.Sc. II Semester Degree Examination
MATHEMATICS
Programming in C
Paper : HCT-2.3

Time : 3 Hours

Maximum Marks :80

Instructions to Candidates:

- 1) Answer any FIVE questions.
- 2) All questions carry equal marks

1. a) Explain the generation of Computers (8)
b) Explain characteristics of G, character Set and G-tokens (8)
2. a) Explain the difference between compiler and an interpreter (8)
b) Explain formatted I/O Statements. (8)
3. a) Explain different types of G-operators with example (8)
b) Explain the evaluation of logical expression, increment, decrement with example. (8)
4. a) Explain If and If-else statement with example. (8)
b) Explain break and continue statements with example. (8)
5. a) Define one-dimensional array and write the Program which accepts n-integers and stores them in an array called num. (8)
b) Write the general form of non-ANSI function definition and write a program which accepts two integers and computes their sum via function addnums(). (8)
6. a) Explain arrays in functions and write a program which illustrates passing of the entire arrays as an argument. (8)
b) Explain static and register variables and write a program which illustrates the working of the external static variables. (8)
7. a) Explain Parameter passing mechanisms and write the program to find the factorial of 4. (8)
b) Explain declaring and initializing string variables. Write the program which accepts a string and counts the number of characters in it and prints the same. (8)

8. a) Explain pointer dereferencing. Write a program which illustrates dereferencing of the content of variables. The variable num is initialized to 400. (8)
- b) Explain call-by reference and write a program which illustrates the call-by-reference method to interchange the contents of two integer variables. (8)
-

PG2S-288-B-23
M.Sc. II Semester Degree Examination
MATHEMATICS
Fuzzy Logic and Applications
Paper - SCT-2.2

Time : 3 Hours

Maximum Marks :80

Instructions to Candidate:

- 1) Answer any FIVE questions.
- 2) All questions Carry equal marks

1. a) Define a fuzzy set and distinguish between fuzzy set and a crisp set. (8)
- b) Define the following.
 - i) Support of a fuzzy set
 - ii) α - cut of a fuzzy set
 - iii) Level set of a fuzzy set
 - iv) Height of a fuzzy set and explain each with a suitable example. (8)
2. a) Show that the De-Morgan's laws are satisfied for the three pairs of fuzzy sets A, B and C with $\mu_A(x) = \frac{1}{1+20x}$, $\mu_B(x) = \left(\frac{1}{1+10x}\right)^{1/2}$ and $\mu_C(x) = \left(\frac{1}{1+10x}\right)^2$
- b) If $A, B \in F(X)$, where $F(X)$ is the fuzzy power set of X , then prove that, the following properties hold for all $\alpha \in [0, 1]$ (8)
 - i) ${}^\alpha(A \cap B) = {}^\alpha A \cap {}^\alpha B$
 - ii) ${}^\alpha(\bar{A}) = (1-\alpha) + \bar{A}$. (8)
3. a) State and Prove, the first decomposition theorem. (8)
- b) If $f: X \rightarrow Y$ be a arbitrary crisp function, then prove that, for any $A \in F(X)$ and $B \in F(Y)$ the following properties of functions obtained by the extension principle hold.
 - i) $f^{-1}(1-B) = 1 - f^{-1}(B)$
 - ii) $f(1-A) = 1 - f(A)$. (8)

4. a) Define Archimedean t-norm and show that, the standard fuzzy intersection is the only idempotent t-norm. (8)

b) Suppose that, a given fuzzy complement C has a unique equilibrium, e_c , then prove that

$$a \leq c(a) \text{ iff } a \leq e_c$$

$$\text{and } a \geq c(a) \text{ iff } a \geq e_c \quad (8)$$

5. a) State and Prove the characterization theorem of t-conorms. (8)

b) Given t-norm i and an involutive fuzzy complement C , then prove that, the binary operation u on $[0, 1]$ defined by $u(a,b) = c(i(c(a), c(b)))$ for all $a, b \in [0, 1]$ is a t-conorm such that $\langle i, u, c \rangle$ is a dual triplet. (8)

6. a) Write a detailed note on aggregation operations of fuzzy sets. (8)

b) Define a fuzzy number and explain with examples. (8)

7. a) Write a note on lattice of fuzzy numbers. (8)

b) Let MIN and MAX be binary operations on \mathbb{R} defined by

$$\mu_{MIN}(A, B)(z) = \sup_{z = \min(x, y)} \min[\mu_A(x), \mu_B(y)]$$

$$\mu_{MAX}(A, B)(z) = \sup_{z = \max(x, y)} \min[\mu_A(x), \mu_B(y)] \text{ for all } z \in \mathbb{R}. \text{ Then prove that, for any}$$

$A, B \in \mathbb{R}$, the following properties hold

i) $MIN(A, B) = MIN(B, A)$

ii) $MAX(A, B) = MAX(B, A)$. (8)

8. a) Write note on fuzzy relation. (8)

b) State the properties of fuzzy relations on a single set and illustrate with examples. (8)

PG2S-285-B-23
M.Sc. II Semester Degree Examination
MATHEMATICS
Algebra - II
Paper - HCT 2.2

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

- 1) *Answer any FIVE questions.*
- 2) *All questions Carry equal marks*

1. a) Define linear dependent and linear independent vectors. Further if v_1, v_2, \dots, v_n are linearly independent in V then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_n v_n, \forall \lambda_i \in F$. (8)
- b) If v_1, v_2, \dots, v_n are in V if a basis of V over F and if w_1, w_2, \dots, w_m in V are linearly independent over a field F then prove that $m \leq n$. (8)
2. a) Let A be an algebra with a unit element over F and suppose that A is of dimensional m over F then prove that every element in A satisfies some non-trivial polynomial in $F[x]$ of degree at most m . (8)
- b) If V is a finite dimensional over F . Then prove that $T \in A(V)$ is singular if and any if there exists $v \neq 0$ in V such that $T(v) = 0$ (8)
3. a) If $\lambda \in F$ is characteristic root of $T \in A(V)$ then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$. (8)
- b) If $\lambda \in F$ is characteristic root of $T \in A(V)$, then show that λ is a root of the minimal polynomial of T . In particular, T has only a finite numbers of characteristic roots in F . (8)
4. a) If $T \in A(V)$ has all its characteristic roots in F then there is a basis of V in which the matrix of T is triangular. (8)
- b) If V is n -dimensional over F and if $T \in A(V)$ has all its characteristic roots in F . Then prove that T satisfies a polynomial of degree n over F . (8)