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PGIS-048-A-22

M.A/M.Sc. I Semester (CBCS) Degree Examination

STATISTICS

Linear Algebra

Paper : HCT - 1.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **SIX** questions from Part- A and **FIVE** questions from Part - B.

PART - A **(6×5=30)**

1. Decide the nature of the following vector & also obtain relationship in case of dependence using sweepout method. X_1 = (1,2,3,3)X_2=(-1,0,4,5)X_3=(0,1,3,4)

2. Explain Gramschemidit's orthogonalization process.

3. Let V={(x,y): y=mx+c; c $\neq$ 0} check whether V is a subspace or not.

4. Show that the number of members in a basis of subspace is invariant.

5. Find the inverse of A by Frames method, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

6. Extend the vectors X_1 = (1,0,1), X_2 = (-1,1,0) to form a basis for V_3.

7. Let A and B be any two square matrices of order nxn then show that $R(AB) \geq R(A) + R(B) - n$

8. Prove that eigne values of a Hermition matrix are real.

PART - B **(5×10=50)**

9. Obtain orthogonal vector by using the vectors (1,0,1), (-1,1,0) and (-3,2,0).

10. a) Obtain the basis for subspace spanned by the vectors X_1=(1,2,3),X_2=(2,3,4), X_3=(3,4,5,), X_4=(4,5,6).

    b) If M_1 and M_2 be any two subspaces having the null vector as the only common vector then prove that dim $(M_1 U M_2) = \dim(M_1) + \dim(M_2)$

11. a) Define Skew Hermitian matrix. Prove that a skew Hermitian matrix reduces to a real skew symmetric matrix where all the elements are real.

b) Define Moore-Penrose inverse. Prove its uniqueness property.

12. a) Find the inverse of matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ by partition method.

b) Find a g-inverse for the following matrix $A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

13. Find characteristic root s and corresponding vectors of a given matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

14. a) Prove that the system AX=0 has n-r linearly independent solutions, where 'n' is the number of columns in A and 'r' is its rank.

b) Show that for any characteristic root λ, G.M. $(\lambda.) \leq$ A.M. (λ).

15. a) Find for what value of η the following system is consistent $x + y + z = 1 : x + 2y + z = \eta : x + 4y + 10z = \eta^2$.

b) State and prove sylvester's law of inertia.

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PGIS-049-A-22

M.Sc. I Semester (CBCS) Degree Examination

STATISTICS

Probability Theory

Paper : HCT - 1.2

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **six** questions from **Part- A** and any **five** questions from **Part - B.**

PART - A

(6×5=30)

1. Define Borel function, simple function and elementary function.
2. Define real valued function, inverse function and indicator function.
3. If $\sum_{i=0}^{n} PA_n < \infty$ then show that P $\left(\overline{\lim} A_n\right) \geq 0$.
4. Show that if EX and EY exist, then E $(X \pm Y) = EX \pm EY$.
5. Define
   i) Mutual Convergence.
   ii) Convergence in probability.
6. Show that if $X_n \xrightarrow{p} X$ and $X \xrightarrow{p} X^1$ then X and X^1 are equivalent.
7. Find the characteristic function of Poisson distribution.
8. State and prove Khintchine's WLLN.

PART - B

(5×10=50)

9. a) Explain briefly the concept of probability.
   b) Show that if $A_n \to A$ then $P(A_n) \to P(A)$.

10. State and prove Holder's inequality.

11. State and prove monotone convergence theorem.

12. Define convergence almost surely and show that $X_n \xrightarrow{r} X \Rightarrow E|X_n|^r \to E|X|^r$

13. Prove any two properties of characteristic function.

14. State and prove inversion formula of a characteristic function.

15. Define WLLN. Discuss Bernoullis and Chebychev's WLLN's.

16. State and prove Liapunov's form of CLT.

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PGIS-050-A-22

M.A./M.Sc I Semester (CBCS) Degree Examination

STATISTICS

Estimation Theory

Paper : HCT - 1.3

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **Six** questions from **Part- A** and **Five** questions from **Part - B.**

PART - A **(6×5=30)**

1. Define unbiasedness , consistency and efficiency of an estimator.
2. Let X_1, X_2, X_3, ... X_n be b (1,P) random variables. Show that $T_n = \sum_{i=1}^{n} X_i$ be sufficient for p.
3. State and prove a sufficient condition of consistency of an estimator.
4. Let X ~ B (N,P) then prove that T = $\overline{X}$ /N is MVB estimator of P.
5. Define the Maximum Likelihood Estimator (MLE) and explain the method of obtaining MLE.
6. Explain the method of minimum Chi-square to find an estimator of parameters of population.
7. Show that $\{P(\lambda), \lambda > 0\}$ is complete family.
8. Define CAN and BAN estimators and give an example of BAN estimator.

PART - B **(5×10=50)**

9. Obtain an unbiased estimator of $\theta^k, k > 0$ in $U(0,\theta)$ based on r^{th} order statistic.
10. a) Show that sample geometric mean is consistent estimator of θ in U(0, θ).

    b) Let T_n be the consistent estimator for g(θ). show that if $T_1 \xrightarrow{p} g_1$ and $T_2 \xrightarrow{p} g_2$ $T_1T_2 \xrightarrow{p} g_1 g_2$ $as\ n \to \infty$

11. Let $N \sim N(\mu, \sigma^2)$. Obtain a sufficient statistic for

i) μ if σ^2 is known

ii) σ^2 if μ is known and

iii) μ and σ^2 if both are unknown.

12. State and prove Crammer-Rao inequality.

13. State and prove Rao -Blackwell theorem.

14. If $x_1, x_2, x_3, ...x_n$ be iid random variables from N (μ, σ^2) then obtain the confidence interval for mean μ at confidence level $1-\alpha$, when σ^2 is known

15. a) Describe the method of moments of estimation by giving an example.

b) If $X \sim P(\lambda)$, find the MLE of λ.

16. Write short note on any **two** of the following :

i) Point estimation

ii) Confidence interval

iii) MVUE

iv) UMA and UMAU confidence sets.