Roll No. ____________________ [Total No. of Pages : 3

PG4S-O-571-B-22

M.Sc. IV Semester Degree Examination

MATHEMATICS

Differential Geometry

Paper : HCT - 4.4

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Solve any* ***Five*** *questions.*
2. *All questions carry* ***equal*** *marks.*

1. a) If f and g are the functions on E^3, v_p and w_p are the tangent vectors, a,b are the numbers, then prove the following:

i) $(av_p + bw_p)[f] = av_p[f] + bw_p[f]$.

ii) $v_p[af + bg] = av_p[f] + bv_p[g]$.

iii) $v_p[fg] = v_p[f].g(p) + f(p).v_p[g]$. **(8)**

b) Define reparametrization of a curve. Find a straight passing through the point $(1,-3,-1)$ *and* $(6,2,1)$. Does this line meet the line passing through points (-1,1,0) and (-5,-1,-1). **(8)**

2. a) If f be a real valued function, ϕ *and* ψ be 1-forms on E^3, then prove the following

i) $d(f\phi) = df \wedge \phi + f\, d\phi$

ii) $d(\phi \wedge \psi) = d\phi \wedge \psi - \phi \wedge d\psi$ **(8)**

b) Define image of a curve under the mapping If the mapping $F: E^2 \rightarrow E^2$ is defined by F (u,v) = ($u^2 v^2$, 2uv) and $\alpha(t) = (r\cos t, r\sin t), 0 \le t < 2\pi$, *is a curve in* E^2 then find the image of the curve α and explain the effect of F on α. **(8)**

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3. a) Establish the Frenet formulae for an unit speed curve $\beta : T \to E^3$ *with* $K > 0$. Compute Frenet frame (T,N,B) of an unit speed curve $\beta(s) = \left(a\cos\frac{s}{c}, a\sin\frac{s}{c}, b\frac{s}{c}\right)$ *where* $c = (a^2 + b^2)^{1/2}$. **(10)**

b) Define a plane curve in E^3. Further, prove that for an unit speed curve β *in* E^3 *with* $K > 0$ is a plane curve if and only if $\tau = 0$. **(6)**

4. a) Define a cylindrical helix. Prove that a regular curve α with K>0 is a cylindrical helix if and only if $\frac{\tau}{K}$ is constant. **(8)**

b) If (E_1, E_2, E_3) be a frame field on E^3 and for each tangent vector v to E^3 at the point p, let $w_{ij}(v) = \Delta_v E_i \; E_j(p), (1 \le i, j \le 3)$ *then show that each* w_{ij} is a 1-form and $w_{ij} = - w_{ji}$. **(8)**

5. a) Define an isometry of E^3. Further, if F is an isometry of E^3 with F(0) = 0, then prove that F is an orthogonal transformation. **(8)**

b) If F is an isometry of E^3, then prove that there exists a unique translation T & α unique orthogonal transformation C such that F = TC. **(8)**

6. a) Define congruence of curves with an example. Two curves $\alpha, \beta : I \to E^3$ are parallel if their velocity vectors $\alpha^1(s)$ *and* $\beta^1(s)$ are parallel for each S in I. If $\alpha(s_0) = \beta(s_0)$ *for some* s_0 *in* I then show that $\alpha \cong \beta$. **(8)**

b) If $\alpha, \beta : I \to E^3$ are unit speed curves such that $k_\alpha = k_\beta$ *and* $\tau_\alpha = \pm\tau_\beta$, then prove that α *and* β are congruent curves. **(8)**

7. a) Define a cylinder in E^3. Further, prove that every cylinder in E^3 is a surface in E^3. **(8)**

b) Prove that a mapping $X : D \to E^3$ is regular iff X_u(d) & X_v(d) are the u, v partial derivatives of X(u,v) = X(d) are linearly independent $\forall \; d \in D$ *where* $D \subset E^3$ **(8)**

8. a) Explain the Stereo graphic projection of the punctured sphere S onto the plane. (6)

b) If ϕ be a 1-form on M and if X & Y are pathces in M, then prove that $d_X\phi = d_Y\phi$ on the overlap of X(D) and Y(D). (10)
