Roll No. ______________________ [Total No. of Pages : 2

PGIIIS-N-037-A-22

M.Sc. III Semester Degree Examination

MATHEMATICS

Functional Analysis

Paper : HCT - 3.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Solve any **FIVE** questions.
2. All questions carry **EQUAL** marks.

1. a) Let (X,d) be a metric space. Then prove that arbitrary union of open sets is open. **(8)**

   b) Let (X,d) be a metric space and A, any subset of X. Then prove that A is closed if and only if its complement (i.e. X-A) is open. **(8)**

2. a) Define the following. **(8)**

   i. Cauchy sequence.

   ii. Convergent sequence.

   iii. Complete metric space.

   iv. Metric space of first and second category.

   b) Show that the set C of complex numbers with usual metric is a complete metric space. **(8)**

3. a) Define the following : **(8)**

   i. Banach space.

   ii. Continuous function.

   iii. Jointly continuous function.

   iv. Continuous linear transformation.

   b) Let N and N^1 be normed linear spaces and T be a linear transformation of N into N^1. Then, prove that T^{-1} exists and is continuous on its domain if and only if there exists a constant k > 0 such that $k\|x\| \le \|T(x)\| \; \forall x \in N$. **(8)**

4. a) Show that the linear space R^n and C^n of all n - tuples $x = (x_1, x_2, ..., x_n)$ of real and complex numbers are Banach spaces under the norm $\|x\| = \left(\sum |x_i|^2\right)^{1/2}$. **(8)**

b) Let on be a linear subspace of a normed linear space N and let f be a functional defined on M. Then, prove that f can be extended to a functional f_0 on the whole space N such that $\|f\| = \|f_0\|$. **(8)**

5. a) Let H be a Hilbert space and x,y be any two vectors of H, then prove the following. **(8)**

i. Parallelogram law $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$.

ii. Polarisation identity

$$4(x, y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2.$$

b) Let M be a closed linear subspace of a Hilbert space H and x be any vector not in M. If d be the distance from x to M, then prove that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$. **(8)**

6. a) State and prove Projection theorem. **(8)**

b) Let y be a fixed vector in a Hilbert space H and let f_y be a scalar valued function on H defind by $f_y(x) = (x, y), \forall x \in H$. Show that f_y is a functional in H*. Also show that $\|y\| = \|f_y\|$. **(8)**

7. a) Define the following. **(8)**

i. Positive operator.

ii. Normal operator.

iii. Unitary operator.

iv. Invariant.

b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **(8)**

8. a) If T is a normal opertaor on a Hilbert space H, then prove that the eigen spaces of T are pairwise orthogonal. **(6)**

b) State and prove spectral theorem. **(10)**

Roll No. ____________________ [Total No. of Pages : 3

PGIIIS-N-041-A-22

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Operations Research - II

Paper - OET - 3.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any five questions.
2. All questions carry **equal** marks.

1. a. Use Big - M method to solve the following L.P.P.

Minimize $Z = 5x_1 + 3x_2$.

Subject to $2x_1 + 4x_2 \le 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \ge 10$

and $x_1, x_2 \ge 0$. **(8)**

b. Use two - phase simplex method to solve the following L.P.P.

Maximize $Z = 5x_1 - 4x_2 + 3x_3$

subject to $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \le 76$

$8x_1 - 3x_2 + 6x_3 \le 50$

and $x_1, x_2, x_3 \ge 0$. **(8)**

2. a. Write the dual of the following L.P.P.

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to $4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

and $x_1, x_2 \geq 0$. (6)

b. Use dual simplex method to solve the following L.P.P. (10)

Minimize $z = 3x_1 + x_2$

Subject to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

and $x_1, x_2 \geq 0$.

3. a. Explain (8)

i. Two - person zero - sum games.

ii. Payoff matrix.

b. Solve the game whose payoff matrix is (8)

Player B

$$\text{Player A} \begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix}$$

4. a. Solve the following game graphically (8)

Player B

$$\textit{Player A} \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

b. Using the principle of dominance, solve the following game (8)

Player B

Player A

	B_1	B_2	B_3	B_4	B_5
A_1	3	5	4	9	6
A_2	5	6	3	7	8
A_3	8	7	9	8	7
A_4	4	2	8	5	3

5. a. Explain a queueing system with its characteristics. (8)

b. Explain departure process. (8)

6. a. Discuss $(M / M / 1):(N / FIFO)$ model. (10)

b. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming poisson arrivals and exponential service distribution, find The steady - state probabilities for the various number of trains in the system, Also find the average waiting time of a new train coming into the yard. (6)

7. a. Write steps in simulation. (8)

b. Discuss event - type simulation. (8)

8. a. Discuss simulation languages. (8)

b. Discuss Monto - Carlo simulation techniques. (8)

Roll No. ______________________ [Total No. of Pages : 2

PGIIIS-N-40-A-22

M.Sc. III Semester Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper : SCT - 3.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **FIVE** questions.
2. All questions carry **equal** marks.

1. a. Show that surfaces exist which cut stream lines orthogonally if the velocity potential exist. (8)

b. Derive the condition for a surface to be a possible form of boundary surface. (8)

2. a. Derive the equation of continuity in orthogonal curvilinear co-ordinates. (8)

b. Prove that liquid motion is possible when velocity at (x,y,z) is given by $u=\frac{3xz}{r^5}, v=\frac{3yz}{r^5};\ w=\frac{3z^2-r^2}{r^5}$ where $r^2=x^2+y^2+z^2$ and the stream lines are the intersection of the surfaces $(x^2+y^2+z^2)^3=C(y^2+z^2)^2$ by the planes passing through ox and also the velocity potential is $\frac{\cos\theta}{r^2}$. (8)

3. a. Derive Helmholtz vorticity equation. (8)

b. Obtain general equations of motion for impulsive action. (8)

4. a. State and prove permanence of irrotational motion. (8)

b. Derive image of a source and doublet in a circle. (8)

5. a. Derive the expression for the general motion of the cylinder. (8)

b. Derive $w=\frac{ik}{2\pi}\log z$. (8)

6. a. A circular cylinder is placed in a uniform stream. Find the forces acting on the cylinder. **(8)**

b. The circle $(x+a)^2+y^2=a^2$ is placed in an on - coming wind of velocity U and there is a circulation $2\pi k$. Find the complex potential and show that the moment about the origion is $2\pi k\,\rho$ aU. **(8)**

7. a. Show that when a cylinder moves uniformly in given straight line in an infinite liquid, the path of any point of the fluid is given by the equation.

$$\frac{dz}{dt}=\frac{Va^2}{(z^1-vt)^2},\ \frac{dz^1}{dt}=\frac{Va^2}{(z-Vt)^2}$$ where $z=x+iy$, $z^1=x-iy$ V=velocity of cylinder. **(8)**

b. Prove that the necessary and sufficient condition for irrotational motion is that there exist a potential ϕ such that $\overline{q}=-\nabla\phi$, q being the velocity vector. **(8)**

8. a. State and prove Kelvins minimum energy theorem. **(8)**

b. A rigid envelope is filled with homogeneous friction less liquid, show that it is not possible by any movements applied to the envelop, to set its contents into motion which will persist after the envelope has come to rest. **(8)**

Roll No. ______________________ **[Total No. of Pages : 2**

PGIIIS-O-045-A-22

M.Sc. III Semester Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper : SCT - 3.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any FIVE questions.
2. All questions carry equal marks.

1. a. Derive equation of continuity by Lagrangian and Eulerian method. **(8)**

b. If the velocity $\overline{q} = xi - yj$, determine the equation of stream line. **(8)**

2. a. Show that the variable ellipsoid **(8)**

$\frac{x^2}{a^2k^2t} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form for the boundary surface of a liquid at any time t.

b. Derive Lambs hydro dynamical equation. **(8)**

3. a. State and prove Kelvins circulation theorem. **(8)**

b. Find the complex potential for a two - dimensional source of strength m placed at the origin. **(8)**

4. a. A region is bounded by a fixed quadrant arc and its radii with a source and an equal sink at the ends of one of the bounding radii. Show that the motion is given by $W = -m\log\left(\frac{z^2 - a^2}{z}\right)$ and prove that the stream line leaving either the source or sink at an angle α with the radius is $r^2\sin(\alpha+\theta) = a^2\sin(\alpha-\theta)$. **(8)**

b. In steady two - dimensional motion given by the complex potential $w = f(z) = \phi + i\psi$, if the pressure thrusts on the fixed cylinder of any shape are represented by a force (X,Y) and a couple of moment N about the origin of co-ordinates, then neglecting external forces show that $X - iY = \frac{i\rho}{2}\int_C \left(\frac{dw}{dz}\right)^2 dz$ and $N = R.P.\left[\frac{-1}{2}\rho\int_C \left(\frac{dw}{dz}\right)^2 z\,dz\right]$ where ρ is the density and integrals are taken round the contour C of the cylinder. (8)

5. a. Find the velocity potential and stream function at any point of a liquid contained between Co-axial cylinders of radii 'a' and 'b' (a<b) when the cylinders are moved suddenly parallel to themselves in directions at right angles with velocities U and V respectively. (8)

b. Discuss the equation of motion of a circular cylinder with circulation. (8)

6. a. Find the pressure on the cylinder for

$$W = U\left(z + \frac{a^2}{z}\right) + \frac{ik}{2\pi}\log z$$ (8)

b. Show that the velocity potential of sphere is $\phi = \left[Ar^n + Br^{-(n+1)}\right]P_n(\mu)$ Where $\mu = \cos\theta$ and (r, θ, w) are the spherical coordinates.. (8)

7. a. State and prove Kutta - Joukowski's theorem. (8)

b. Determine the stagnation points for $\frac{\partial\phi}{\partial r} = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta$. (8)

8. a. Derive vorticity transport equation. (8)

b. Derive the necessary and sufficient condition that vertex lines may be at right angles to the stream lines. (8)