Roll No. ____________________ [Total No. of Pages : 4

PG2S-065-B-22
M.Sc. II Semester Degree Examination
MATHEMATICS
Operations Research - I
Paper : OET - 2.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) Answer any **Five** questions.
2) **All** questions carry **equal** Marks.

1. a) Define Operations Research (O.R.). What is its Scope? Explain briefly. **(8)**

b) A City hospital has the following minimal daily requirements for nurses:

Period	Clock time (24 hours day)	Minimal numbers of nurses required
1	6AM - 10AM	2
2	10AM - 2PM	7
3	2PM - 6PM	15
4	6PM - 10PM	8
5	10PM - 2AM	20
6	2AM - 6AM	6

Nurses report to the hospital at the begining of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate the problem as a linear programming problem. **(8)**

2. a) Use graphical method to solve the following L.P.P.

Minimize $Z = 20x_1 + 40x_2$

Subject to $36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$20x_1 + 10x_2 \geq 100$

and $x_1, x_2 \geq 0$ **(8)**

b) Solve the following L.P.P by Simplex method.

Maximize $Z = -x_1 + 3x_2 - 2x_3$

Subject to $3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

and $x_1, x_2, x_3 \geq 0$ **(8)**

3. a) Explain the method of row minima and matrix minima to obtain initial basic feasible solution to the given transportation problem. **(10)**

b) Determine the initial basic feasible solution to the following transportation problem by using North-West corner rule.

(Destinations)

(Origins)	A	B	C	(Availability)
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
(Requirements)	4	2	2	

(6)

4. a) Explain the method of column minima to obtain an initial basic feasbile solution to the given transportation problem. **(4)**

b) Find the optimum transportation schedule for the following problem.

(Destinations)

(Source)		D_1	D_2	D_3	D_4	(Supply)
	O_1	19	30	50	10	7
	O_2	70	30	40	60	9
	O_3	40	8	40	20	18
(Demand)		5	8	7	14	

(12)

5. a) Write the Hungarian method of assignment. (8)

b) Solve the following assignment problem.

	A	B	C	D
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

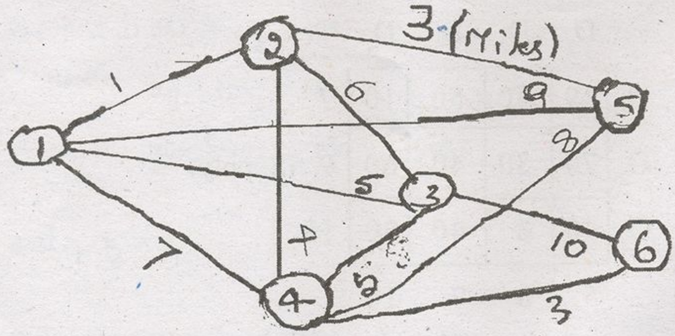
(8)

6. a) Given the following matrix of set up costs, show how to sequence production so as to minimize set cost per cycle.

	A	B	C	D	E
A	∞	4	7	3	4
B	4	∞	6	3	4
C	7	6	∞	7	5
D	3	3	7	∞	7
E	4	4	5	7	∞

(8)

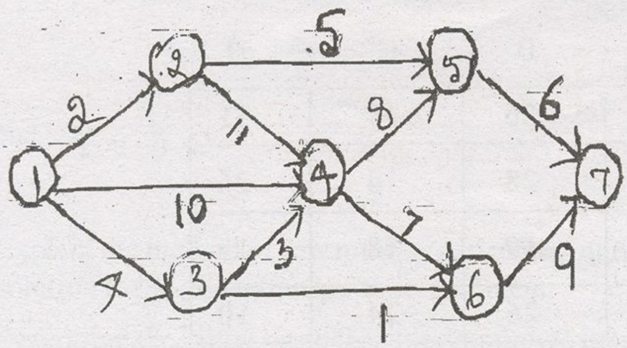
b) The Midwest T.V. cable Company is in the process of providing cable services to five new housing development areas. The figure below depicts the potential T.V linkages among the five areas. The cable miles are shown on each branch.



Determine the most economical cable network for the Midwest Company. **(8)**

7. a) Write the Dijkstra's shortest Path algorithm. **(8)**

b) For the following network, find the shortest route from node 1 to node 7.



(8)

8. a) Explain branch and bound method. **(8)**

b) Use branch and bound method to solve the following L.P.P.

Minimize $Z = 4x_1 + 3x_2$

Subject to $5x_1 + 3x_2 \geq 30$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1, x_2 \geq 0$ and are integers. **(8)**

Roll No. ______________________ Total No. of Pages : 3

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PG2S-064-B-22
M.Sc. II Semester Degree Examination
MATHEMATICS
Complex Analysis
Paper : SCT2.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) Answer any **Five** questions.
2) **All** questions carry **equal** Marks.

1. a) State and Prove the necessary and sufficient conditions for the derivative of the function $f(z)=u(x, y)+iv(x,y)$ in a given region R. **(8)**

b) Evaluate $I=\int_0^{2+i} \bar{z}^2 dz$

i) Along the line $y=x/2$ and

ii) Along the real axis to 2 and then vertically to $(2+i)$ **(8)**

2. a) Let *f(z)* be analytic everywhere within and on a simple closed contour C, taken in the positive sense. If z_0 is any point interior to C, then prove that

$$f(z_0)=\frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{(z-z_0)}$$ **(10)**

b) Evaluate

$$I=\int_C \frac{e^{2z}}{(z+1)^4}dz$$

Where C is the circle $|Z|=1$. **(6)**

3. a) Prove that, a sequence $Z_n = x_n + iy_n$ (n = 1, 2,......) is convergent if and only if the two real sequences $\{x_n\}$ and $\{y_n\}$ are convergent. **(8)**

b) State and Prove Wierstrass M-test. **(8)**

4. a) Let *f(z)* is an analytic function throughout a disk $|z-a| < R$ where R is the radius and a is the centre. Then prove that, *f(z)* has the power series representation

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n, \; |z-a| < R \text{ where } a_n = \frac{f^{(n)}(a)}{n!}, \; n = 0, 1, 2, \ldots\ldots \quad \textbf{(10)}$$

b) Find the Poles of the function

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3 (3z+2)^2}$$ and also determine its order. **(6)**

5. a) State and Prove Cauchy's Residue theorem. **(8)**

b) If *f(z)* has a pole of order *m* at $z = z_0$, then prove that the residue at $z = z_0$ is given by

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{1}{(m-1)!} \lim_{z \to z_0} \frac{d^{m-1}}{dz^m} [(z-z_0)^m f(z)]$$ **(4)**

c) Find the residue of the function

$$f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$$ at z=1, 2 & 3. **(4)**

6. a) Let a function *f(z)* is analytic except for isolated singular points in extended complex plane. Then prove that

the sum of all the residues of *f(z)* is zero. **(8)**

b) Evaluate $I = \int_0^{2\pi} \frac{Cosn\theta}{1 + 2aCos\theta + a^2} d\theta$ by using calculus of residues. **(8)**

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7. a) Evaluate $I = \int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ **(8)**

b) Using the contour integration Prove that

$$\int_0^{\infty} \frac{Cosmx}{(a^2+x^2)}dx = \frac{\pi}{2a}e^{-ma}$$ **(8)**

8. a) State and Prove argument principle. **(8)**

b) State and Prove Mittag-Leffler theorem: **(8)**
