Roll No. ______________________ [Total No. of Pages : 2

PGIS-032-A-22

M.Sc. I Semester Degree Examination

MATHEMATICS

Real Analysis

Paper : HCT - 1.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** questions.
2. **All** questions carry **equal** marks.

1. a. Define Riemann integral, Refinement and Riemann - stieltjes integral. (8)

   b. A function f is integrable with respect to α on [a,b] if and only if for every $\in > 0$ there exists a partition p of [a,b] such that (8)

   $U(p, f, \alpha) - L(p, f, \alpha) < \in$.

2. a. If $f \in \mathbb{R}(\alpha_1)$ and $f \in \mathbb{R}(\alpha_2)$ then $f \in \mathbb{R}(\alpha_1 + \alpha_2)$ and $\int_a^b fd(\alpha_1 + \alpha_2) = \int_a^b fd\alpha_1 + \int_a^b fd\alpha_2$ and if $f \in \mathbb{R}(\alpha)$ and C a +ve constant, then $f \in \mathbb{R}(C\alpha)$ and $\int_a^b fd(\alpha) = C\int_a^b fd\alpha$. (8)

   b. Suppose $f \in \mathbb{R}(\alpha)$ on [a,b] $m \le f \le M$ ϕ is continuos on [m, M] and $h(x) = \phi[f(x)]$ on [a,b] Then $h \in \mathbb{R}(\alpha)$ on [a,b]. (8)

3. a. If $f \in R$ on [a,b] and if there is a differentiable function F on [a,b] such that $F' = f$ then $\int_a^b f(x)dx = F(b) - F(a)$. (8)

   b. Show that the series $\sum a_n(x) g_n(x)$ will be uniformly convergent on [a,b] if (8)

      1. The sequence $< g_n(x) >$ is a positive monotonic decreasing sequence converging uniformly to zero for all $x \in [a,b]$.

      2. $|f_n(x)| = \left|\sum_{r=1}^{n} a_r(x)\right| < k$, $\forall x \in [a,b]$ and $\forall n \in N$, k is a fixed number independent of x.

4. a. Test for the uniform convergence the series $\sum_{n=0}^{n-1} x e^{-nx}$ in the interval [0,1]. **(8)**

b. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E. Then show that $\{f_n\}$ has a sequence $\{f \cap k\}$ such that $\{f \cap k(x)\}$ converges for every $x \in E$. **(8)**

5. a. State and prove Abel's theorem. **(8)**

b. Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\ldots \; -1 < x \leq 1$, and deduce that,

$\log(x) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots..$ **(8)**

6. a. Prove that there exists a positive number π such that $C(\pi/2) = 0$ and $C(x) > 0$, for $0 \leq x \leq \pi/2$, where π is the smallest positive root of the equation C(x) = 0. **(8)**

b. If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its fourier coefficient, then $\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$ converges. **(8)**

7. a. Find the Fourier series generated by the periodic function $|x|$ of period 2π, Also compute the value of series at $0, 2\pi, -3\pi$. **(8)**

b. A linear operator T on a finite dimensional vector space X is One - to - One if and only if the range of T is all of X. **(8)**

8. State and prove Rank - Theorem. **(16)**

4. a. Test for the uniform convergence the series $\sum_{n=0}^{n-1} x e^{-nx}$ in the interval [0,1]. **(8)**

b. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E. Then show that $\{f_n\}$ has a sequence $\{f \cap k\}$ such that $\{f \cap k(x)\}$ converges for every $x \in E$. **(8)**

5. a. State and prove Abel's theorem. **(8)**

b. Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\ldots$ $-1 < x \leq 1$, and deduce that,

$\log(x) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots..$ **(8)**

6. a. Prove that there exists a positive number π such that $C(\pi/2) = 0$ and $C(x) > 0$, for $0 \leq x \leq \pi/2$, where π is the smallest positive root of the equation $C(x) = 0$. **(8)**

b. If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its fourier coefficient, then $\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$ converges. **(8)**

7. a. Find the Fourier series generated by the periodic function $|x|$ of period 2π, Also compute the value of series at $0, 2\pi, -3\pi$. **(8)**

b. A linear operator T on a finite dimensional vector space X is One - to - One if and only if the range of T is all of X. **(8)**

8. State and prove Rank - Theorem. **(16)**

Roll No. ______________________ [Total No. of Pages : 2

PGIS-033-A-22

M.Sc. I Semester Degree Examination

MATHEMATICS

Algebra - I

Paper - HCT - 1.2

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a. Define Centraliser, centre of a group, conjugate element. Prove that the relation of conjugacy is an equivalence relation in G. **(8)**

b. Prove that every quotient group of a cyclic group is cyclic but not convergency. **(8)**

2. a. If p is a prime number and G is a non - abelian group of order p^3, show that the centre of G has exactly P elements. **(8)**

b. Suppose H and K are subgroups of a finite group G. Also let $O(H) > \sqrt{O(G)}$, $O(K) > \sqrt{O(G)}$ then show that $H \cap K = \{e\}$. **(8)**

3. a. Define external and internal direct products. **(8)**

b. State and prove cauchy theorem for finite Abelian group. **(8)**

4. a. State and prove Third sylow theorem. **(8)**

b. State and prove Scheiers theorem. **(8)**

5. a. Define ring, Euclidean domain. Let R be a euclidean domain. Then show that $a \in R$ s a unit iff d(a) = d(1). **(8)**

b. State and prove Unique Eactorisation domain. **(8)**

6. a. Let P be a prime integer and suppose that for some integer C relatively prime to P and for the integers x and y $x^2 + y^2 = cp$ then show that p can be written as the sum of squares of two integers a,b such that $p = a^2 + b^2$. **(8)**

b. State and prove FERMAT theorem. **(8)**

7. a. If F is a field, then show that F[x] is a Euclidean domain. (8)

b. Let R be a unique factorisation Domain. Then the product of two primitive polynomials over R is also a primitive polynomial. (8)

8. a. State and prove Gauss theorem. (8)

b. Define R - module submodule and isomorphism of R - module. (8)

Roll No. ____________________ [Total No. of Pages : 2

PGIS-034-A-22

M.Sc. I Semester Degree Examination

MATHEMATICS

Ordinary Differential Equations

Paper : HCT - 1.3

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Solve any **Five** questions.
2. All questions carry **EQUAL** marks.

1. a) For any real x_0 and constants α, β prove that there exists a solution ϕ of the initial value problem $L(y) = y'' + a_1 y' + a_2 y = 0$, with $y(x_0) = \alpha$, $y'(x_0) = \beta$ in $-\infty < x < \infty$. (8)

b) If $\phi_1, \phi_2, \ldots\ldots, \phi_n$ are n solutions of L(y) = 0 on I, then show that they are linearly independent iff $W(\phi_1, \phi_2, \ldots\ldots \phi_n) \neq 0 \ \forall x$. (8)

2. a) If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are any n constants and x_0 be any real number, then there exists a solution ϕ of $L(y) = y'' + a_1 y' + a_2 y = 0$ on $-\infty < x < \infty$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, \ldots\ldots, \phi(n-1)(x_0) = \alpha_n$. (8)

b) Let $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be n linearly independent solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on I. If ϕ is any solution of L(y) = 0 then it can be written in the form $\phi = C_1\phi_1 + C_2\phi_2 + \ldots\ldots + C_n\phi_n$, where C_1, C_2, ..., C_n are constants. (8)

3. a) Define adjoint and self adjoint equation. Further transform the following equations into equivalent self - adjoint equation and write the equation

i. $x^2 y'' - 2xy' + 2y = 0$.

ii. $f(x)y'' + g(x)y' = 0$. (8)

b) State and prove sturm separation theorem. (8)

4. a) State and prove sturm comparison theorem. (8)

b) Show that the zeros of the functions $a\sin x + b\cos x$ and $c\sin x + d\cos x$ are distinct and occur alternately whenever $ad - bc \neq 0$. (8)

5. a) Define ordinary and singular points, Further, explain the method of solving second order linear differential equation for which $x = 0$ is an ordinary point. **(8)**

b) Solve $x(x-1)y''+(3x-1)y'+y=0$ by Frobenius method. **(8)**

6. a) Define the following :

i. Orthogonal set of functions.

ii. Orthonormal set of functions.

iii. Orthonormal set of functions with respect to a weight functions. **(8)**

b) Find the power series solution of the initial value problem $xy''+y'+2y=0$, in powers of (x-1) with initial conditions $y(0)=2$, $y'(0)=4$. **(8)**

7. a) Explain Gram - schmidt process of orthonormalization. **(8)**

b) Show that the functions $f_1(x)=4$, $f_2(x)=x^3$ are orthogonal on the interval (-2,2) and determine constants A and B so that the functions $f_3(x)=1+AX+BX^2$ is orthogonal to both f_1 and f_2. **(8)**

8. a) Prove that eigen functions correspondings to different eigen values are orthogonal with respect to some weight function. **(8)**

b) Find the eigen vaues and the corresponding eigen functions of $X''+\lambda X=0$, with $X(0)=0$ and $X'(L)=0$ **(8)**

Roll No. ____________________ [Total No. of Pages : 2

PGIS-035-A-22

M.Sc. I Semester Degree Examination

MATHEMATICS

Discrete Mathematics

Paper - HCT - 1.4

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a) What is propositional logic? Discus existential quantification and universal quantification of a predicate with suitable examples . **(4)**

b) Define **(6)**

i) Tautology.

ii) Contradiction and discuss various forms of tautologies.

c) Determine whether the following argument is logically correct. **(6)**

If I work hard and I have talent, then I will get a good job

If I get a good job, then I will be happy

$\therefore$ I will be not happy, then I did not work hard

or i did not have a talent.

2. a) Show by mathematical induction that any finite non empty set is countable. **(4)**

b) Suppose that a valid computer password consists of seven characters, the first of which is a letter chosen from the set {A,B,C,D,E,F,G} and the remaining six characters are letters chosen from the english alphabet or a digit. How many different passwords are possible? **(6)**

c) If S and T are any two finite sets and $S \cap T = \phi$, then prove that $|S \cup T| = |S| + |T|$. **(6)**

3. a) Write the numeric function corresponding to the generating functions.

i. $A(Z) = \dfrac{2+3Z-6Z^2}{(1-2Z)}$.

ii. $A(Z) = \dfrac{Z^4}{(1-2Z)}$. **(4)**

b) Solve the recurrence relation $a_r = a_{r-1} + a_{r-2}$. (6)

c) Solve the recurrence relation $4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$. (6)

4. a) Solve

$a_r + a_{r-1} = 3r2^r$. (6)

b) Solve $a_r + 6a_{r-1} + 9a_{r-2} = 3$ with $a_0 = 1$ and $a_1 = 1$. (6)

c) Describe the method of solution of recurrence relations by the method of generating functions. (4)

5. a) Define the following and explain with examples. (6)

i. Reflexive relation.

ii. Symmetric relation.

iii. Asymmetric relation.

iv. Transitive relation.

b) Let L be a lattice, then for every a and b in L prove that

i. $a \vee b = b$ iff $a \leq b$.

ii. $a \wedge b = a$ iff $a \vee b = b$. (10)

6. a) If the join operation is distributive over the meet operation, then prove that, the meet operation is distributive over the join operation. (6)

b) Let $(A, \vee, \wedge, -)$ be a finite boolean algebra. Let b be any non zero element in A and $a_1, a_2, \ldots, a_k$ be all the atoms of A such that $a_i \leq b$, then prove that $b = a_1 \vee a_2 \vee \ldots \vee a_k$. (6)

c) Write a note on switching circuits. (4)

7. a) Prove that every group of order 4 is abelian. (4)

b) Prove that, the set of all even permutations on a set S forms a group with respect to composition of permutations. (6)

c) Prove that the relation $a \equiv b \bmod H$ is an equivalence relation. (6)

8. a) Describe the process of coding of binary information and error detection. (6)

b) Find the weight of each of the following words in B^5.

(i) x = 10000 (ii) x = 11100 (iii) x = 11111 (iv) x = 00000. (4)

c) Prove that, an encoding function $e : B^m \to B^n$ can detect k or fewer errors if and only if its minimum distance is at least (K+1). (6)
