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PGIVS-O-039 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

(Measure Theory)

Paper : HCT - 4.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a) Define outer measure of a set. Let X be a space of atleast two points and $x_0 \in X$. for each $A \subset X$, define $\mu(A)=\begin{cases}0 \text{ if } x_0 \notin A \\ 1 \text{ if } x_0 \in A\end{cases}$ then prove that μ is an outer measure. **(8)**

b) Prove that an outermeasure is monotonic and σ - subadditive. **(8)**

2. a) Show that an open set in a metric space is measurable with respect to any outer measure. **(8)**

b) Show that $m_e(A) \geq m_i(A)$ for any set A, where $m_e(A)$ is the exterior measure and $m_i(A)$ is the interior measure of a Set A. **(8)**

3. a) If A and B are any two measurable sets such that $A \subset B$ then show that $m_i(A) \leq m_i(B)$. **(8)**

b) State and prove the first fundamental theorem. **(8)**

4. a) If f is a measurable function defined over a measurable set E. Let 'C' is any real number then show that $cf, f+c, |f|, f^2$ are measurable functions. **(8)**

b) Define a characteristic function of a set. Show that the characteristic function of a set E is measurable if and only if E is a measurable set. **(8)**

5. a) State and prove the Lebesgue bounded convergence theorem. **(10)**

b) If two bounded measurable functions f and g are equivalent on a measurable set E i.e., if f = g a.e. on E then prove that they have the same integral. **(6)**

6. a) If a function f is absolutely continuous in an interval and if $f'(x) = 0$ almost everywhere then show that f is constant. **(8)**

b) Define absolutely continuous function. If f(x) and g(x) are absolutely continuous functions then prove that their product and quotient are also absolutely continuous. **(8)**

7. a) If f is an integrable function on [a,b] and if $F(x) = \int_a^x f(t)dt + F(a)$ then prove that $F'(x) = f(x)$ a.e. on [a,b]. **(8)**

b) Show that every absolutely continuous function is an indefinite integral of its own derivative. **(8)**

8. a) Show that the union of any countable collection of positive subsets of X is positive. **(8)**

b) State and prove Lebesgue Decomposition theorem. **(8)**

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PGIVS-N-038 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

Measure Theory

Paper - HCT - 4.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a) Define an outer measure of a set. If X be a space of atleast two points and $x_0 \in X$. for each $A \subset X$, defined $\mu(A) = \begin{cases} 0 \text{ } if \text{ } x_0 \in A \\ 1 \text{ } if \text{ } x_0 \in A \end{cases}$ then prove that μ is an outer measure. **(8)**

b) Prove that the union and Intersection of two outermeasurable sets is outermeasurable. **(8)**

2. a) Show that an open set in a matric space is measurable with respect to any outer measure. **(8)**

b) Define exterior and interior measure of a set. Show that $m_e(A) \geq m_i(A)$ for any set A, where $m_e(A)$ and $m_i(A)$ are the exterior and interior measure of A respectively. **(8)**

3. a) If E_1, E_2, ... are disjoint measurable sets and $E = E_1 + E_2 + ...$ then prove that E is measurable and $m(E) = \sum_{k=1}^{\infty} m(E_k)$. **(8)**

b) Define the set of the type F_σ and G_σ. Show that the sets of the type F_σ, G_σ and the Borel sets are measurable. **(8)**

4. a) If f is a measurable functions defined over a measurable set E. Let 'C' be any real number then show that $cf, f+c, |f|, f^2$ are measurable functions. **(8)**

b) Let f be a function defined on a measurable set E then prove that f is measurable iff for any open set $G \subset \mathbb{R}$, $f^{-1}(G)$ is a measurable set. **(8)**

5. a) Let <f_n> be a sequence of functions which converges in measure to the function f on a measurable set E. Then prove that there exists a subsequence which also converges to the function f almost everywhere. **(8)**

b) Let <f_n> be a sequence of bounded measurable functions defined over a measurable set E and the sequence <f_n> converge in measure to a measurable function f on the set E then show that $\lim_{n\to\infty}\int_E f_n(x)dx = \int_E f(x)dx$. **(8)**

6. a) State and prove Beppo - Levi's theorem. **(10)**

b) Using the Lebesgue's dominated convergence theorem, evaluate the following integral,

$\lim_{n\to\infty}\int_0^1 f_n(x)dx$ where $f_n(x) = \frac{n^{3/2}x}{1+n^2x^2}, 0 \le x \le 1$, n = 1,2,3,........ **(6)**

7. a) If f(x) and g(x) are absolutely continuous functions, then prove that their sum, difference and product are also absolutely continuous functions. **(6)**

b) Define an indefinite integral of a function. Prove that an indefinite integral is an absolutely continuous function. **(10)**

8. a) Define a positive set. Show that a union of any countable collection of positive subsets of X is positive. **(6)**

b) Let (X, A, μ) be a σ - finite measure space and ν be a σ - finite measure defined on A. Then prove that there exists two uniquely determined measures ν_0 and ν_1 such that $\nu = \nu_0 + \nu_1$ and $\nu_0 \perp \mu$, $\nu_1 << \mu$. **(10)**

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PGIVS-N-040-B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

Graph Theory - II

Paper - HCT 4.2

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any FIVE full questions.
2. All questions carry Equal marks.

1. a) Define a minimal dominating set with an example. If G is a graph with no isolated vertices then prove that the complement V-S of every minimal dominating set S is a dominating set. **(8)**

   b) Prove that for any tree T with $P \geq 2$, there exists a vertex $v \in V$ such that $\gamma(T-v)-\gamma(T)$. **(8)**

2. a) Define diameter of a graph with an example. For any connected graph G, Prove that $\left\lceil \frac{diam(G)}{3}+1 \right\rceil \leq \gamma(G)$. **(8)**

   b) Prove that for any graph G, $\gamma(G)+\gamma(\overline{G}) \leq P+1$ and $\gamma(G)\gamma(\overline{G}) \leq P$. **(8)**

3. a) Prove that every maximal independent set in G is a minimal dominating set of G. **(8)**

   b) Show that for any graph G with $n \geq 3$ vertices then $\gamma_t(G) \leq \frac{2n}{3}$. **(8)**

4. a) Prove that if G be a connected graph of order n containing a cycle then $\gamma_r(G) = n-2$ if and only if G is C_4 or C_5 can be obtained from C_5 by attaching zero or more leaves to atmost two of the vertices of the cycle. **(8)**

   b) Prove that if T is a tree on two or more vertices then $\gamma_R(T) = \gamma(T)+1$ if and only if T is a wounded spider. **(8)**

5. a) Show that a graph is the line graph of a tree if and only if it is a connected block graph in which each cutvertex is on exactly two blocks. **(8)**

b) Prove that the second line graph $L^2(G)$ of a finite connected graph G is minimally non outerplaner if and only if G satisfies the following conditions.

i. $\deg v \le 3$ for every vertex v of G.

ii. $\deg u + \deg v \le 5$ for every edge uv of G.

iii. G has exactly one cycle and it contains exactly one vertex of degree 3. **(8)**

6. a) Define total block graph of a graph. Prove that the total - block graph $T_B(G)$ of a graph G is outerplanar if and only if each component of G is a path. **(8)**

b) Prove that the lict graph n(G) of a graph G is planar if and only if G is planar and the degree of each vertex is atmost 3. **(8)**

7. a) Prove that lict graph n(G) and litact graph m(G) of a graph G are isomorphic if and only if G has atmost one cutvertex. **(8)**

b) Define total graph of a graph. Prove that the total graph T(G) of a graph G is planar if and only if $\Delta(G) \le 3$ and every vertex of degree 3 is a cutvertex. **(8)**

8. a) Define the crossing number. Prove that the total - block graph $T_B(G)$ of any nonplanar graph G has crossing number atleast 3. **(8)**

b) Let G be a graph with $\Delta \ge 3$ and if $\Delta(G) = 3$ has a non cutvertex of degree 3. Then show that G has a total graph with crossing number 1 if and only if it has no subgraph homeomorphic to $K_{1,5}$, $K_4 - x$, $K_2 \circ \overline{K}_3$ or $2K_{1,4}$ or $K_1 + K_2 \circ \overline{K}_2$. **(8)**

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PGIVS-N-42 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

(Computational Numerical Methods - II)

Paper - HCT - 4.3

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a) Use Runge - Kutta fourth order method to compute y(0.2) for the initial value problem $y' = -2xy^2, y(0) = 1$ and h = 0.1 **(8)**

b) Solve simultaneous differential equations $\frac{dy}{dx} = z, \frac{dz}{dx} = xz - y$ in $0 \le x \le 0.2$ with h = 0, y(0) = 3 and z(0) = 0 by Runge - Kutta method of fourth order. **(8)**

2. a) Solve $y'' = xy' - y$, y(0) = 3, $y'(0) = 0$ using Runge - Kutta fourth order method for x = 0.1 with h = 0.1 **(8)**

b) Derive Adam's - Bash forth predictor corrector formula to solve an initial value problem. **(8)**

3. a) Use Milne's predictor corrector method to compute y(0.4) of the initial value problem $y' = x + y^2$, y(0) = 1 with h = 0.1. **(8)**

b) Solve the boundary value problem $y'' + y + 1 = 0, y(0) = 0$ and y(1) = 0. **(8)**

4. a) Describe the explicit finite difference scheme for solving a general parabolic partial differential equation. **(8)**

b) Find the solution of IBVP

$u_t = u_{xx} + (x-2)u_x - 3u$ with conditions $u(x,0) = x^2 - 4x + 5$, $0 \le x \le 4$, t = 0; u (0,t) = u(4,t) = $5e^{-t}$, t> 0 using explict finite difference scheme. **(8)**

5. a) Solve the following equation using Crank - Nicolson method.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ with the conditions } u(x,0) = \begin{cases} 2x, & 0 \le x \le 0.5 \\ 2(1-x), & 0.5 \le x \le 1 \end{cases}$$

u(0,t) = u(1,t) = 0. **(8)**

b) Discuss ADI method for parabolic partial differential equation. **(8)**

6. a) Derive standard five point formula and diagonal five point formula for solving laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. **(8)**

b) Solve $u_{xx} + u_{yy} = 0$ subject to boundary conditions $u(0,y) = 0$; $u(x,0) = 0$; $u(x,1) = 100x$; $u(1,y) = 100y$ for square region with h = 0.25. **(8)**

7. a) Discuss implicit method for hyperbolic partial differential equation. **(8)**

b) Solve $u_{tt} + u_{xx} = 0$ for t > 0, 0<x<1 by finite difference method with initial conditions

$$(t = 0) \quad u = \begin{cases} 5(x-0.3); 0.3 < x < 0.5 \\ 5(0.7-x); 0.5 < x < 0.7 \\ 0; \textit{for all other points} \end{cases}$$

$\frac{\partial u}{\partial t} = 0$ at t = 0 and boundary conditions u(0,t) = 0 and u (1,t) = 0. **(8)**

8. a) Discuss explicit method for hyperbolic partial differential equation. **(8)**

b) Derive the formula for solving poisson's equation $u_{xx} + u_{yy} = f(x,y)$. **(8)**

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PGIVS-O-043 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

(Computational Numerical Methods - II)

Paper - HCT 4.3

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a) Use Runge - Kutta fourth order method to compute y(0.2) for the initial value problem $y' = x + y^2$, y(0) = 1 and h = 0.1. **(8)**

b) Solve $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y^2 = 0$ using Runge - Kutta fourth order method for x = 0.2 with initial conditions y(0) = 1, $y'(0) = 0$. **(8)**

2. a) Derive Adam - Bashforth predictor - corrector formula to solve an initial value problem. **(8)**

b) Use Adam's - Bashforth predictor corrector method to compute y(0.8) of the initial value problem $y' = 1 + y^2$, y(0) = 0 with h = 0.2. **(8)**

3. a) Use Milne's predictor corrector method to find y(0.3) from $y' = x + y^2$, with y(0) = 1.**(8)**

b) Using shooting method, solve $y'' = x + y$; y(0) = 0, y(1) = 0, $0 \le x \le 1$ with initial slope once with $y'(0) = 1/2$ and another $y'(0) = -1/2$ and step size 0.5. Use Taylor's series for obtaining solution. **(8)**

4. a) Describe briefly the classification of a partial differential equation as :

i. Elliptic

ii. Hyperbolic

iii. Parabolic types and give one example for each type. **(8)**

b) Describe the explicit finite difference scheme for solving parabolic partial differential equation. **(8)**

5. a) Find the numerical solution of IBVP given by $u_t = u_{xx} + (x-2)u_x - 3u$, with Initial conditions : $u(x,0) = x^2 - 4x + 5$, $0 \le x \le 4$, t > 0 Boundary conditions : $u(0,t) = u(4,t) = 5e^{-t}, t > 0$ using implicit difference scheme. **(8)**

b) Solve by Crank - Nicholson scheme $u_{xx} = u_t$, satisfying the conditions u(x,0) = 0, u(0,t) = 0, u(1,t) = t for two time steps by choosig h = 0.25. **(8)**

6. a) Discuss ADI method for parabolic partial differential equation. **(8)**

b) Solve $u_{xx} + u_{yy} = 0$ with boundary values $u(0,y) = 0; u(x,0) = 0;$ $u(x,1) = 100x$, $u(1,y) = 100y$, on a square region using grid spacing h = 0.25. Apply Gauss - Seidel iteration, process to improve the solution at the interior mesh points. **(8)**

7. a) Discuss implicit method for hyperbolic partial differential equation. **(8)**

b) Find the solution of the I.B.V.P.

$u_{xx} - u_{yy} = 0$ with the conditions

$u(x,0) = x(1-x); 0 \le x \le 1$

$u_y(x,0) = 1;\ 0 \le x \le 1$

u(0,y) = y(1-y) = u(1,y) for y > 0 using implicit method. **(8)**

8. a) Discuss explicit method for hyperbolic partial differential equation. **(8)**

b) Solve $u_{tt} + u_{xx}$ for t > 0, 0<x<1 by finite difference method with initial conditions

$$(t = 0) \quad u = \begin{cases} 5(x-0.3); 0.3 < x < 0.5 \\ 5(0.7-x); 0.5 < x < 0.7 \\ 0;\ for\ all\ other\ points \end{cases}$$

$\frac{\partial u}{\partial t} = 0$ at t = 0 and boundary conditions u(0,t) = 0 and u (1,t) = 0.. **(8)**

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PGIVS-N-044 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

Differential Geometry

Paper : HCT - 4.4

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Solve any five questions.
2. All questions carry **equal** marks.

1. a) Define the derivative mapping of a mapping. Find $F_*(V_p)$ where $V=(2,-1,3)$, $P=\left(2,\frac{\pi}{2},\pi\right)$ and the mapping F is defined by, $F(x,y,z)=(x\cos y, x\sin y, z)$. **(8)**

b) Define a curve in E^3 show that the curves given by $(t,1+t^2,t)$ and $(\sin t,\cos t,t)$ have the same initial velocity V_p If $f=x^2-y^2+z^2$ then compute $V_p[f]$ by evaluating f on each of there curves. **(8)**

2. a) What is an wedge product simplify the following forms

i. $d[fdg+gdf]$,

ii. $[(f-g)(df+dg)]$

iii. $d[fdg\wedge gdf]$

iv. $(gfdf)+d(fdg)$ **(8)**

b) Define image of a curve under the mapping. If the mapping $F:E^2\to E^2$ is defined by $F(u,v)=(u^2-v^2,2uv)$ & $\alpha(t)=(r\cos t, r\sin t)$, $0\le t\le 2\pi$ is a curve in E^2 then find the image of the curve α and explain the effect of F on α. **(8)**

3. a) If (e_1, e_2, e_3) be a frame at a point p of E^3. If v is any tangent vector to E^3 at E^3 at p then derive the orthogonal expansion about $v$as $v = (v.e_1)e_1 + (v.e_2)e_2 + (v.e_3)e_3$. **(8)**

b) Define a plane curve in E^3. Prove that for an unit speed curve β in E^3 with K>0 is a plane curve if & only if $\tau = 0$. **(8)**

4. a) Define a cylindrical helix in E^3. Prove that a regular curve α, with K>0 is a cylindrical helix if & only if the ratio $\frac{\tau}{K}$ is constant. **(8)**

b) Compute Frenet apparatus K, τ, T, N, B of the unit speed curve. $P = \left(\frac{4}{5}\cos s, 1 - \sin x, \frac{-3}{5}\cos s\right)$. **(8)**

5. a) Define isometry and orthogonal transformation of E^3. If F is an isometry of E^3 such that $F(\nu) = 0$ then prove that F is an orthogonal transformation. **(8)**

b) If F is an isometry of E^3 then prove that there exists a unique translation T & a unique orthogonal transformation C such that $F = TC'$. **(8)**

6. a) Define sing of an isometry show that all translations and all rotations are orientation preserving. **(8)**

b) Define congruence of curves with an example Two curves $\alpha, \beta : I \to E^3$ are parallel if their velocity vectors $\alpha'(s)$ & $\beta'(s)$ are parallel for each s in I. If $\alpha(s_0) = \beta(s_0)$ for some s_0 in I then show that $\alpha = \beta$. **(8)**

7. a) Prove that every sphere in E^3 is an surface in E^3. Further, defind a simple surface in E^3. **(8)**

b) Prove that a mapping $X : D \to E^3$ is regular iff $X_u(d)$ & $X_v(d)$ are the u,v partial derivatives of $X(u,v) = X(d)$ are linearly independent $\forall d \in D$, where $D \subset E^2$. **(8)**

8. a) Explain the stereo graphic projection of the punctured sphere is S onto the plane. **(8)**

b) If ϕ be a 1-form on M and If X & Y are patches in M then prove that $d_x\phi = d_y\phi$ on the overlap of x(D) and y(E). **(8)**

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PGIVS-N-046 B-21

M.Sc. IV Semester Degree Examination

MATHEMATICS

Fluid Mechanics - II

Paper - SCT - 4.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any five questions.
2. All questions carry **equal** marks.

1. Derive equation of conservation of energy. **(16)**

2. a) Prove that any problem can be expressed in the form of the relationship between a group of variables ie, $f(V_1, V_2, \ldots\ldots\ldots V_i \ldots\ldots\ldots V_n) = 0$ where f denotes a function of some variables $V_i = (i = 1, 2 \ldots\ldots\ldots n)$. **(8)**

   b) Define Froude number pressure coefficient, Mach number and Reynolds number. **(8)**

3. a) Define Lift coefficient, coefficient of heat transfer, local skin friction coefficient and peclet number. **(8)**

   b) Prove that there are only five independent dimensionless groups in the compressible fluid motion. **(8)**

4. a) The discharge through a horizontal capillary tube depends upon the drop per unit length the diameter, and the viscosity. Find the form of the equation. **(8)**

   b) The loss of pressure ΔP for laminar flow in a pope is a function of pipe length l, diameter D, mean velocity U and the dynamic viscosity μ. Determine the expression for the pressure loss. **(8)**

5. a) Derive the skin friction at the plates for couette flow. **(8)**

   b) A 10 inch thick wall holds water at 50°F on one side. At a depth of 12 inch below the surface of water, 0.015 inch crack has developed on the wall for a length of 24 feet. Determine (i) the average velocity, the mass rate of flow throught the crack, and the maximum velocity in the crack (ii) the Reynolds number of the flow based on the average velocity and the width of the crack (iii) the shear - stress distribution. **(8)**

6. a) Derive the temperature distribtution for plane couette flow and discuss the physical significance of Nussell - number. **(8)**

b) Derive the flux $Q = \frac{\pi C}{4\mu} \frac{a^3 b^3}{a^2 + b^2}$ for the fluid over the area of the elliptic cross - section. **(8)**

7. a) Derive the velocity distribution u(y,t) = U erfn η for the unsteady motion over a flat plate. **(8)**

b) Derive the shearing stress for unsteady flow of viscous incompressible fluid between two parallel plates. **(8)**

8. a) Derive momentum integral equation for the boundary layer. **(8)**

b) Determine the local frictional coefficient for flow over a flat plate based on the Von karman's integral equation. **(8)**
