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PGIIIS-O-834 A-21
M.Sc. III Semester (CBCS) Degree Examination
MATHEMATICS
Operations Research - II
Paper : OET 3.1
(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) Explain Big - M method for solving L.P.P. **(8)**

b) Use two - phase Simplex method to solve the following L.P.P.

Maximize $Z = x_1 + 2x_2 + 3x_3$

Subject to $x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_1 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$. **(8)**

2. a) Write the dual of the following L.P.P.

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$. **(6)**

b) Solve the following L.P.P by dual simplex method.

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

Subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$. **(10)**

3. a) Explain

i) Two Person Zero - sum games.

ii) The Maximin Minimax principle. **(8)**

b) Define a Saddle point. Solve the game whose payoff matrix is given by

Player B

$$\text{Player A} \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array} .$$ **(8)**

4. a) Solve the following 2×3 game graphically

Player B

$$\text{Player A} \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \end{array} .$$ **(8)**

b) Using the principle of dominance, solve the following game

Player B

$$\text{Player A} \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} \end{array} .$$ **(8)**

5. a) Explain a queueing system with its characteristics. **(8)**

b) Explain pure birth process. **(8)**

6. a) Discuss $(M/M/1):(\infty/FIFO)$ model. **(8)**

b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter - arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the followings.

i) The mean queue size (in length).

ii) The probability that the queue size exceeds 10.

iii) If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii). **(8)**

7. a) Write the steps in Simulation. **(8)**

b) Discuss event - type Simulation. **(8)**

8. a) Discuss the advantages and limitations of Simulation. **(8)**

b) Customers arrive at a milk booth for the required service. Assume that inter arrival and service time are constants and given by 1.8 and 4 time units respectively. Simulate the system by hand computation for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility? **(8)**

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PGIIIS-N-834 A-21
M.A./M.Sc. III Semester (CBCS) Degree Examination
MATHEMATICS
Operations Research - II
Paper : OET 3.1
(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any* ***FIVE*** *questions.*
2) *All questions carry* ***equal*** *marks.*

1. a) Use Big - M method to solve the following L.P.P. **(8)**

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

and $x_1, x_2, x_3, x_4 \geq 0$.

b) Use two - phase simplex method to solve the following L.P.P. **(8)**

Maximize $Z = 5x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$

and $x_1, x_2 \geq 0$.

2. a) Write the dual of the following L.P.P. **(6)**

Minimize $Z = 4x_1 + 6x_2 + 18x_3$

Subject to $x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$

and $x_1, x_2, x_3 \geq 0$.

b) Use dual simplex method to solve the following L.P.P. **(10)**

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

and $x_1, x_2 \geq 0$.

3. a) Determine which of the following two - person zero - sum games are strictly determinable and fair. Give optimum strategies for each player in the case of strictly determinable games: **(8)**

i) $$\text{Player A} \overset{Player B}{\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}}$$

ii) $$\text{Player A} \overset{Player B}{\begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix}}$$

b) Solve the game whose payoff matrix is **(8)**

$$\text{Player A} \overset{Player B}{\begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix}}.$$

4. a) Solve the following 2×3 game graphically **(8)**

$$\text{Player A} \overset{Player B}{\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}}.$$

b) Using the principle of dominance, solve the following game **(8)**

$$\text{Player A} \overset{Player B}{\begin{bmatrix} 9 & 2 \\ 8 & 6 \\ 6 & 4 \end{bmatrix}}.$$

5. a) Define a queueing system and explain in brief the characteristics of the queueing system. **(8)**

b) Explain pure birth process. **(8)**

6. a) Discuss $(M / M / 1):(N / FIFO)$ model. **(8)**

b) Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter - arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purposes), calculate the probability that the yard is empty and find the average queue length. **(8)**

7. a) Explain the concept of even - type simulation. **(8)**

b) Customers arrive at a milk booth for the required service. Assume that inter - arrival and service times are constant and given by 1.8 and 4 time units respectively. Simulate the system by hand computations for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility? (Assume that the system starts at t=0). **(8)**

8. a) Discuss Monte - Carlo Simulation Technique. **(8)**

b) Discuss Simulation languages. **(8)**

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PGIIIS-N-830 A-21
M.Sc. III Semester Degree Examination
MATHEMATICS
Functional Analysis
Paper : HCT 3.1
(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **FIVE** questions.*
2) *All questions carry **equal** marks.*

1. a) State and prove cantor's intersection theorem. **(8)**

b) State and prove Bair's category theorem. **(8)**

2. a) If X is a complete metric space and Y is a subspace of X, then prove that Y is complete if & only if it is closed. **(8)**

b) Prove the following: **(8)**

i) The union of any number of open sets in X is open.

ii) The intersection of a finite number of open sets in X is open.

3. a) State and prove Uniform boundedness theorem. **(8)**

b) State and prove Riesz's lemma. **(8)**

4. a) Prove that a product of normed linear space X of X_1 and X_2 is complete if and only if X_1 and X_2 are complete. **(8)**

b) State and prove Banach steinhaus theorem. **(8)**

5. a) If M is a closed linear subspace of a normed linear space and x_0 is a vector not in M, then show that there exists a functional f_0 in N^* such that $f_0(M)=0$ & $f_0(x_0)\neq 0$. **(8)**

b) State and prove Hahn - Banach theorem. **(8)**

6. a) State and prove closed graph theorem. **(8)**

b) Prove that a Banach space is a Hilbert space iff the parallelogram law holds. **(8)**

7. a) State and prove projection theorem. **(8)**

b) Show that the adjoint of T is a unique continuous linear operator on H with the definition $\langle T_x, y \rangle = \langle x, T^* y \rangle$. (8)

8. a) If H is a Hilbert space and f an arbitrary function in H^* then prove that there exists a unique vector y in H such that $f(x) = (x, y),\ \forall\ x \in H$. **(8)**

b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then show that the linear subspace $M + N$ is also closed. **(8)**

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PGIIIS-O-833 A-21

M.Sc. III Semester Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper : SCT 3.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any* ***FIVE*** *Questions.*
2) *All Questions carry* ***Equal*** *marks.*

1. a) Define Stream line, Path line, Streak line, Vorticity vector and boundary surface. **(8)**

b) Derive equation of continuity by Lagrangian method. **(8)**

2. a) Show that $u=\dfrac{-2xyz}{\left(x^2+y^2\right)^2}$, $v=\dfrac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}$, $w=\dfrac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational. **(8)**

b) Show that $\phi=(x-t)(y-t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the stream lines at time 't' are the curves $(x-t)^2-(y-t)^2=$ constant and that the paths of fluid particles have the equations $\log(x-y)=\frac{1}{2}\left\{(x+y)-a(x-y)^{-1}\right\}+b$ where a and b are constants. **(8)**

3. a) Prove that the difference of the values of ψ at the two points represents the flux of a fluid across any curve joining the two points. **(8)**

b) Find the image of a simple source with respect to a straight line and show that the image of a doublet with respect to a plane is an equal doublet symmetrically placed. **(8)**

4. a) What arrangements of sources and sinks will give rise to the function $w = \log\left(z - \frac{a^2}{z}\right)$?

Draw a rough sketch of the stream lines in this case and prove that two of them subdivide into a circle $r = a$ and axis of y. **(8)**

b) State and prove Milne Thomson circle theorem. **(8)**

5. a) Find the velocity potential and stream function at any point of a liquid contained between co - axial cylinders of radii a and b $(a < b)$ when the cylinders are moved suddenly parallel to themselves in direction at right angles with velocities U and V respectively. **(8)**

b) Discuss the equation of motion of a circular cylinder with circulation. **(8)**

6. a) State and prove Blasius theorem. **(8)**

b) A circular cylinder is fixed across the stream of velocity U with circulation K around the cylinder. Show that the maximum velocity in the liquid is $2U + \frac{K}{2\pi a}$ where a is the radius of the cylinder. **(8)**

7. a) Show that the fluid pressure exerts a force $\frac{\dot{M}\dot{U}}{2}$ opposing the motion. **(8)**

b) State and prove Kutta - Joukowski's theorem. **(8)**

8. a) Derive the necessary and sufficient condition that vortex lines may be at right angles to the stream lines. **(8)**

b) State and prove Euler's momentum theorem. **(8)**

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PGIIIS-O-831 A-21
M.Sc. III Semester Degree Examination
MATHEMATICS
Graph Theory - I
Paper : HCT 3.2
(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **Five** full questions.*
2) *All questions carry **equal** marks.*

1. a) Prove that every u-v walk contains a u-v path. (5)

   b) Define a cut vertex and a bridge in a connected graph. Draw a graph with more cut vertices than bridges. (6)

   c) Construct two cubic graphs with 6 and 8 vertices. (5)

2. a) Show that for any graph G with six vertices G or $\overline{G}$ contains a cycle. Illustrate through an example. (6)

   b) Show that a graph G is bipartite if and only if all its cycles are even. (5)

   c) Draw all r - regular graphs, $0 \leq r \leq 5$ with 6 vertices. (5)

3. a) Define a tree. Prove that a (p,q) graph is a tree if and only if it is acyclic and $p = q + 1$. (8)

   b) Show that every nontrivial tree contains at least two end vertices. (8)

4. a) Define eccentricity, radius and diameter of a graph. Show that every tree has either one or two centers. (8)

   b) Prove that in every network, the value of a maximum flow equals the capacity of a minimum cut. (8)

5. a) Define a vertex and an edge connectivity of a graph with an example. Prove that in any graph, $K(G) \leq \lambda(G) \leq \delta(G)$. (8)

   b) Let G be a nontrivial connected graph. Prove that G contains an Eulerian trial if and only if G has exactly two odd vertices. (8)

6. a) Find under what condition the complete bipartite graph $K_{m,n}$ has an Eulerian graph. **(8)**

b) Prove that for every non trivial connected Eulerian graph G, the set of edges of G can be partitioned into cycles. **(8)**

7. a) If G is a connected graph with $p \geq 3$ vertices which is not Hamiltonian then show that the length k of longest path of G satisfies $k \geq 2\delta(G)$. **(8)**

b) Draw the following graphs:

i) An Eulerian graph which is not Hamiltonian.

ii) An Eulerian graph which is also Hamiltonian.

iii) A Hamiltonian graph which is not Eulerian.

iv) A graph which is neither Eulerian nor Hamiltonian. **(8)**

8. a) Show that a graph is the line graph of tree if and only if it is a connected block graph in which each cut vertex lies on exactly two blocks. **(8)**

b) Prove that every tournament has a spanning path. Explain with an example. **(8)**

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PGIIIS-O-832 A-21
M.Sc. III Semester Degree Examination
MATHEMATICS
Computational Numerical Methods - I
Paper : HCT 3.3
(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **FIVE** questions.*
2) *All questions carry **equal** marks.*

1. a) Derive Hermite interpolation formula. **(8)**

   b) Construct Hermite interpolation polynomial that fits the following data. **(8)**

x	0	1	2
$f(x)$	4	-6	-22
$f'(x)$	-5	-14	-17

2. a) Define piecewise linear interpolation and derive the formula for the error in the piecewise linear interpolation. **(8)**

   b) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data: **(8)**

x	1	2	4	8
$f(x)$	3	7	21	73

   Hence estimate the value of f(3) and f(7)

3. a) Describe Newton - Raphson method for solving the system of two non - linear equations in two unknowns. **(8)**

b) Apply Newton - Raphson method to solve **(8)**

$x^2 - y^2 = 3$

$x^2 + y^2 = 13$

Take $(x_0, y_0) = (3,2)$ and perform three iterations.

4. a) Describe Mulers method. **(8)**

b) Describe Graeffe's root squaring method. **(8)**

5. a) Describe Gauss - elimination method of solving linear system of equations. **(8)**

b) Solve the following system of equations by Gauss - elimination method **(8)**

$5x - y - 2z = 142$

$x - 3y - z = -30$

$2x - y - 3z = 5.$

6. a) Describe Gauss - Seidal iteration method of solving system of linear equations. **(8)**

b) Solve the system of equations **(8)**

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$ by Gauss - Seidal iteration method.

7. a) Prove that the iteration method of the form $X^{(k+1)} = HX^{(k)} + C$, $k = 0,1,2,.....$ for the solution of linear system $AX = B$ converges to the exact solution for any initial vector if $\|H\| < 1$. **(8)**

b) Describe SOR method for the solution of linear system of equations. **(8)**

8. a) Describe givens method of finding eigen values and eigenvectors of the real symmetric matrix. **(8)**

b) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tridiagonal matrix form by givens method and find eigen values and eigen vector corresponding to the largest eigen value. **(8)**
