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PGIS-N-230 B-19

M.A./M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

Real Analysis

Paper : HCT 1.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a) If P* is a refinement of P, then prove that

$L(P^*, f, \alpha) \geq L(P, f, \alpha)$ *and* $U(P^*, f, \alpha) \leq U(P, f, \alpha)$ **(8)**

b) If $f \in \mathbb{R}(\alpha_1)$ *and* $f \in \mathbb{R}(\alpha_2)$ then prove that

$f \in \mathbb{R}(\alpha_1 + \alpha_2)$ *and* $\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$ **(8)**

2. a) If f and φ are continuous on [a,b], φ is strictly increasing on [a,b] and χ is inverse function of φ. Then show that $\int_a^b f(x)\, dx = \int_{\varphi(a)}^{\varphi(b)} f(\chi(y))\, d\chi(y)$ **(8)**

b) If Γ is a smooth curve in R^3 such that Γ' exists and is continuous on [a,b] then prove that Γ is rectifiable and has a length $\int_a^b \left|\Gamma'(t)\right| dt$ **(8)**

3. a) If a series $\sum f_n$ converges uniformly to f in [a,b] and x_0 is a point in [a,b] such that $\lim_{n \to \infty} f_n(x) = a_n$, $n = 1, 2, \ldots\ldots$ then prove

i) The series $\sum a_n$ converges and

ii) $\lim_{x \to x_0} f(x) = \sum_{n=1}^{\infty} a_n$ **(8)**

b) Prove that if a series $\sum f_n$ converges uniformly to f in [a,b] and its terms f_n are continuous at a point $x_0 \in [a,b]$, then the sum function f is also continuous at x_0 **(8)**

4. a) If a sequence $\langle f_n \rangle$ of continuous function defined on [a,b] is monotonic increasing and converges to a continuous function f, then show that the convergence is uniform on [a,b]. **(8)**

b) Prove that if f is a bounded variation on [a,b], then it is also bounded variation on [a,c] and [c,b] where c is a point of [a,b] and conversely. Also prove $V(f,a,b)=V(f,a,c)+V(f,c,b)$. **(8)**

5. Show that IA be an algebra of real continuous functions on a complex set K. If IA separates points on K and if IA vanishes at no point of K, then the uniform closure IB of IA consists of all real continuous functions on K. **(16)**

6. a) Prove that if $\sum a_n x^n$ be a power series with finite radius of convergence R and let $f(x)=\sum a_n x^n, -R<x<R,$. If the series $\sum a_n x^n$ converges, then $\lim_{x \to R-0} f(x)=\sum a_n R^n$ **(8)**

b) If a function φ is bounded and integrable on [a,b], then prove that as $n \to \infty$ $A_n = \int_a^b \varphi \cos nx\, dx$ and $B_n = \int_a^b \varphi \sin nx\, dx$ **(8)**

7. a) If a function f is bounded and integrable in [0,a], a>0 and monoton in $]0,\delta[$, $0<\delta<a$, then show that $\lim_{n\to\infty} \int_0^a f \frac{\sin nx}{x} dx = f(0+)\int_0^\infty \frac{\sin x}{x} dx$ **(8)**

b) Define gamma function and prove $\Gamma(\frac{1}{2})=\sqrt{\pi}$. Also find

i) $\Gamma(\frac{3}{2})$ ii) $\Gamma(\frac{5}{2})$ iii) $\Gamma(\frac{7}{2})$ **(8)**

8. a) Show that a linear operation Ton a finite dimensional vector space X is one-to-one if and only if the range of T is all of X. **(8)**

b) Prove that if *f* is a function of class C, then f is a differentiable function. **(8)**

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PGIS-236 B-19

M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

Fuzzy Sets And Fuzzy Systems

Paper : SCT 1.2

(Old and New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **Five** full questions.*
2) ***All** questions carry **equal** marks*

1. a) Define the following **(8)**

i) Fuzzy Set

ii) α-cut of a fuzzy set

iii) Strong α-cut of a fuzzy set

iv) Height of a fuzzy set.

Explain with suitable example.

b) Let $x = \{5,10,20,30,40,50,60,70,80\}$ be the universal set of ages. Adult and young are the two fuzzy sets defined on x as **(8)**

Adult = $0.8/20+1/30+1/40+1/50+1/60+1/70+1/80$

Young = $1/5+1/10+0.8/20+0.5/30+0.2/40+0.1/50$ then find

i) Young $\cup$ Adult

ii) Young $\cap$ Adult

iii) $(\overline{\text{Young} \cup \text{Adult}})$

2. a) Show that the Demorgans laws are satisfied for the three pairs of fuzzy sets A, B, and C with $\mu_A(x)=\frac{1}{1+20x}; \mu_B(x)=\left(\frac{1}{1+10x}\right)^{1/2}; \mu_C(x)=\left(\frac{1}{1+10x}\right)^{2}$. **(8)**

b) Let $A, B \in F(x)$. Then prove the following for all $\alpha \in [0,1]$ **(8)**

i) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$

ii) ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

3. a) For any $A \in F(x)$, prove that ${}^{\alpha+}A = \underset{\alpha<\beta}{U}\, {}^{\beta}A = \underset{\alpha<\beta}{U}\, {}^{\beta+}A$ **(8)**

b) State and prove the first decomposition theorem. **(8)**

4. a) Let $f: X \to Y$ be an arbitrary crisp function. Then for $A \in F(X)$ prove that **(8)**

$\overset{-1}{f}(1-A) \geq 1 - f(A)$

b) Let $C:[0,1] \to [0,1]$ satisfy the axioms C_2 and C_4 of fuzzy complement, then prove that C also satisfies C_1 and C_3 and also show that C is a bijective function. **(8)**

5. a) State and prove second characterization theorem. **(8)**

b) For all $a, b \in [0,1]$, prove that $i_{\min}(a,b) \leq i(a,b) \leq \min(a,b)$ where $i_{\min}$ denotes the drastic intersection. **(8)**

6. a) Given an involutive fuzzy complement C and an increasing generator g of C, then prove that, the t-norm and t-conorm generated by g are dual with respect to C. **(8)**

b) Write a note on fuzzy aggregation operations. **(8)**

7. Let $A \in F(\mathbb{R})$ then, prove that A is a fuzzy number iff there exists a closed interval $[a,b] \neq \phi$ such that
$$\mu_A(x) = \begin{cases} 1 & for \quad x \in [a,b] \\ l(x) & for \quad x \in (-\infty, a) \\ r(x) & for \quad x \in (b, \infty) \end{cases}$$ **(16)**

Where $l:(-\infty,a) \to [0,1]$ is monotonic increasing, continuous from right and such that $l(x)=0$ for $x \in (-\infty, w_1)$; and $r:(b,\infty) \to [0,1]$ is monotonic decreasing, continuous from left and such that $r(x)=0$ for $x \in (w_2, \infty)$.

8. a) Distinguish between fuzzy relations and crisp relations. Give an example of each. **(7)**

b) Define the following **(9)**

i) Height of a fuzzy relation

ii) Inverse of a fuzzy relation

iii) Standard composition

and explain with suitable example.

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PGIS-O-231 B-19
M.Sc. I Semester Degree Examination
MATHEMATICS
Algebra - I
Paper : HCT 1.2
(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. Answer any **five** questions.
2. **All** questions carry **equal** marks.

1. a) Define normalizer of an element. Show that if $a \in G$, G is a group then $N(a)$, normalizer of a is a subgroup of G. **(8)**

b) Show that S_n is a finite group of order $\lfloor n$ and is non-abelian if $n > 2$. **(8)**

2. a) Show that every permutation $\sigma \in S_n$ where S_n is the symmetric group can be expressed as a product of disjoint cycles. **(8)**

b) Derive the class equation for finite group. **(8)**

3. a) State and prove Cauchy's theorem. **(8)**

b) Define a Sylow's P-subgroup of a group G with an example. **(8)**

4. a) Prove that G is solvable iff $G^{(n)} = \{e\}$ for some non-negative integer n. **(8)**

b) Prove that any group of order pq, p and q being distinct primes is solvable. **(8)**

5. a) Show that ring of integers is a Euclidean ring. **(8)**

b) State and prove Eisenstein's criterion of irreducibility. **(8)**

6. a) Show that the set $R[x]$ of all polynomials over R forms a ring for the operations addition and multiplication. **(8)**

b) State and prove first isomorphism theorem for rings. **(8)**

7. a) Show that the product of two primitive polynomials over UFD is a primitive polynomial. (8)

b) Let K/F be a finite extension. Then show that K/F is an algebraic extension. (8)

8. a) Let $f(x) \in F[x]$ be of degree n. Then show that $f(x)$ has a splitting field. (8)

b) Define perfect field. Let F be a field of characteristic $P(\neq 0)$. Show that an element a, in some extension of F, is separable over F iff $F(a^P) = F(a)$. (8)