PGIVS - 1520 A - 16

M.A./M.Sc. IVth Semester (CBCS) Degree Examination

Statistics

(Practical Based on SCT 4.1) Paper: SCP 4.1

Time: 2 Hours

Maximum Marks: 30

Instructions:

Answer any two questions.

1. Solve the following LPP using two-Phase method.

Max
$$Z=12x_1+18x_2+15x_3$$

sub.to $4x_1+8x_2+6x_3 \ge 64$
 $3x_1+6x_2+12x_3 \ge 96$
and $x_1,x_2,x_3 \ge 0$

2. Solve the following QPP using Beals method.

Mini Z=6-6
$$x_1$$
+2 x_1 ²-2 x_1 x₂+2 x_2 ²
Sub.to x_1 + x_2 ≤2
and x_1 , x_2 ≥0

3. Solve the following IPP using Gomorian Cutting Plane method.

Max
$$Z=5x_1+8x_2$$

Sub to $x_1+2x_2 \le 8$
 $4x_1+x_2 \le 10$
and x_1,x_2 integers and non-negative.

4. A small maintenance project consists of the following 12 jobs.

Jobs	Duration in days		
1-2	2		
2-3	7		
2-4	3		

3-4	3
3-5	5
4-6	3
5-8	5
6-7	8
6-10	4
7-9	4
8-9	1
9-10	7

- i) Draw a network for this project.
- ii) Summarise CPM Calculates in a tabular from by calculating total float and hence determine critical path.

PGIVS 1518 A-16 M.A/M.Sc. IVth Semester (CBCS) Degree Examination Statistics

(Practical Based on HCT 4.1) Paper: HCP - 4.1

Time: 2 Hours Maximum Marks: 30

Instructions to Candidates:

Answer any Two questions

1. The total area under apple cultivation in 25 districts is 1000 area. A random sample of 6 villages was selected using PPSWR scheme. The area under apple cultivation and the number of apple bearing trees in the sample villages are given below

Village No.	Area under apple cultivation	No. of bearing trees
1	30	200
2	10	600
3	10	700
4	20	800
5 .	40	700
6	40	600

Estimate the total number of apple bearing trees in the 25 villages. Also find an estimates of the sampling various of the estimator. (15)

2. A district has 500 apple bearing trees. They wire divided into clusters of 4 trees each. A random samples of 5 clustering was selected using SRSWOR scheme. The yield in kg in the sample clusters is given below.

Clusters No.	Trees			
	1	2	3	4
	5.53	4.8	1.69	15.8
2	54.21	34.63	53.7	37.2
3	38	47.07	19.64	29.13
4	6.4	11.68	40.21	6.52

Estimate the total yield in the village and also estimates the S.E of the estimator. (15)

3. At an experimental station there were 100 fields sown with wheat. Each field was divided into 16 plots of equal size out of 100 fields 5 were selected by SRSWOR, scheme. From each selected field, two plots were selected by SRSWOR scheme. The yields in kg/plot are given below.

Selected field	Plots	
	1	2
1	4.32	3.84
2	4.16	4.36
3	3.06	4.24
4	4	4.84
5	4.1	4.68

Estimate the total yield for the experimental station along with the standard error of the estimator. (15)

- 4. Using the data of question no. 1 estimator the total number of apple bearing trees in the 25 villages by
 - i) Ratio method
 - ii) Regression method

Compare the two estimators by estimating their mean squre errors. (15)

PGIVS-1515 A-16 M.A./M.Sc. IVth Semester (CBCS) Degree Examination Statistics

(Sampling Theory)
Paper: HCT - 4.1

Time: 3 Hours Maximum Marks: 80

Instructions to Candidates:

Answer any six questions from Part A and five full questions from Part - B.

Part-A

Answer any six questions.

 $(6 \times 5 = 30)$

- 1. Explain the cumulative total method of selecting a PPS sample.
- 2. Define selection probabilities π_i and π_j ($i \neq j$) and compare them for PP SWOR when sample size is 2.
- 3. With usual notations, prove that $\sum_{i=1}^{N} \pi_i = n$ and $\sum_{\substack{j=1 \ j \neq i}}^{N} \pi_{ij} = (n-1)\pi_i$
- 4. Outline the merits and demerits of cluster sampling.
- 5. Define intra class correlation co-efficient. Obtain the bounds for it in case of equal clusters.
- **6.** Define two phase sampling. When is it useful?
- 7. Discuss optimum allocation in two stage sampling when the precision is fixed.
- 8. Discuss when ratio and regression estimators are equally efficient.

Part - B

Answer any five full questions:

 $(5 \times 10 = 50)$

9. Obtain Horwitz - Thompson estimator $\hat{\gamma}_{HT}$ of the population total under PPSWOR sampling with usual notations, prove that

$$V(\hat{Y}_{HT}) = \sum_{i=1}^{n} \frac{(1-\pi_{i})}{\pi_{i}} Y_{i}^{2} + \sum_{\substack{i=1\\i\neq j}}^{N} \sum_{j=1}^{N} \frac{(\pi_{ij} - \pi_{i}\pi_{j})}{\pi_{i}\pi_{j}} Y_{i}Y_{j}$$

- 10. Explain Midzuno Sen sampling scheme. Find the probability of selecting a given sample under this scheme. Obtain Lahiri unbiased ratio estimator of the population total under this scheme.
- 11. What is two stage sampling? For SRSWOR as sample design at both stages, obtain an unbiased estimator of the population total and obtain its variance.
- 12. Find an unbiased estimator of the population mean when SRSWOR is used in the first phase and PPSWR in the second phase.
- 13. When all the clusters are of the same size, obtain the efficiency of CSRWOR over SRSWOR.
- 14. Obtain the conditions under which CSRWOR is preferable to SRSWOR.
- 15. a) Obtain an estimator of the mean square error of regression estimator of the population mean in SRSWOR scheme.
 - b) Derive exact expression for bias of ratio estimator. (5+5)
- 16. Write short notes on any two of the following:
 - a) Des Raj estimator
 - b) Ratio and regression estimator
 - c) Almost unbiased ratio estimator
 - d) Sampling and non sampling error. $(2\times5=10)$

PGIVS 1516 A-16 M.A./M.Sc. IVth Semester (CBCS) Degree Examination Statistics

(Multivariate Analysis) Paper: HCT-4.2

Time: 3 Hours

Maximum Marks: 80

Instructions to Candidates:

Answer any six questions from Part-A and five from Part-B

PART-A

 $(6 \times 5 = 30)$

- 1. Given $X=Az+\alpha$, where X follows $N_p(\mu, \Sigma)$ distribution, A is a real non-singular matrix and α is a real vector, obtain the distribution of z.
- 2. Define Wishart matrix. Obtain its characteristic function.
- 3. Define Hotelling's T²-statistic. Mention its applications.
- 4. State the hypothesis of symmetry and explain how it is tested?
- 5. Derive a test procedure for testing the hypothesis about the mean vector of normal population when the covariance matrix is known.
- 6. State the maximum likelihood estimators (MLEs) of μ and Σ of $N_p(\mu, \Sigma)$ distribution. Are these estimators unbiased? Justify.
- 7. What are canonical correlations? Give a real life example
- 8. Discuss a method of estimating the probabilities of misclassification.

PART-B

- 9. Let X follows $N_p(\mu, \Sigma)$ distribution, where Σ is a positive definite matrix. Obtain the distribution of
 - i) AX, where A is a non-singular matrix.
 - ii) $(X, \mu) \Sigma^{-1}(X \mu)$

- 10. Show that the marginal distribution obtained from a multivariate is a multivariate normal.
- 11. Obtain MLEs of μ and σ^2 in the basis of random sample from $N_p(\mu, \sigma^2 I)$ distribution.
- 12. Explain how Hotelling's T² is related to F-distribution.
- Derive likelihood ratio test for testing $H_0: \Sigma = \Sigma_0$ on the basis of a random sample from $N_p(\mu, \Sigma)$ distribution.
- 14 Derive Fisher's linear discriminant rule for two normal populations.
- 15 Outline one-way multivariate analysis of variance.
- Derive the first two principal components of a random vector with zero means and covariance matrix Σ

PGIVS 1517 A-16

M.A./M.Sc. IVth Semester (CBCS) Degree Examination Statistics (Optional)

(Operations Research)

Paper : SCT - 4.1(a)

Time: 3 Hours

Maximum Marks: 80

Instructions to Candidates:

Answer any six questions from Part A and five full questions from Part - B.

Part - A

 $(6 \times 5 = 30)$

- 1. Define the following.
 - i) Feasible solution
 - ii) Infeasible solution, and
 - iii) Unbounded solution.
- 2. Show that if the primal has an unbounded solution, the corresponding dual has no feasible solution.
- 3. Define linear programming problem (lpp). Give an example and mathematical form of lpp.
- 4. Explain Lagrangean method of solving NLPP
- **5.** Define integer programming problem (IPP). Explain Gomorian cutting plane method of solving IPP.
- **6.** State the transportation problem. Explain stepping stone method for solving transportation problem.
- 7. Discuss the maximal flow method of solving network problem.
- 8. Explain various costs involved in an inventory problem.

Part - B

 $(5 \times 10 = 50)$

- 9. Show that if at any stage of simplex procedure we have $C_j z_j \ge 0$ for some $a_j \in A$ and $y_j \le 0$, $i = 1, 2, \dots$ then the lpp has an unbounded solution.
- 10. Solve the following lpp using simplex method.

$$\max z = 7x_1 + 5x_2$$

$$subject \ to \ x_1 + 2x_2 \le 6$$

$$4x_1 + 3x_2 \le 12$$

$$x_1, x_2 \ge 0$$

11. Solve the following non linear programming problem:

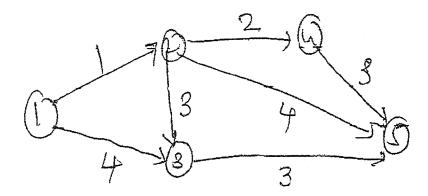
optimize
$$z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

subject to $x_1 + x_2 + x_3 = 15$
 $2x_1 - x_2 + 2x_3 = 20$
 $x_1, x_2, x_3 \ge 0$

- 12. Explain Beale's method for solving a quadratic programming problem.
- 13. Solve the following transformation problem using MODI method

	D_{i}	D_2	D_3	D_4	a_{i}
O_1	9	7	11	10	35
O_2	10	13	14	8	50
O_3	15	10	17	6	40
b_j	45	20	30	30	

14. Find the shortest route from node 1 to each of the other nodes for the following network problem.



- 15. Discuss inventory system with price break.
- 16. Write short notes on any two of the following:
 - a) Charn's M technique of solving lpp.
 - b) U V method of solving transportation problem.
 - c) PERT method of solving Network problem.
 - d) Discuss Hasse's model of Inventory system.

PGIVS 1519 A-16 M.A/MSc IV Semester (CBCS) Degree Examination STATISTICS

(Practical Based on HCT 4.2) Paper: HCT-4.2

Time: 2 Hours

Maximum Marks: 30

Instruction to Candidates: Answer any two questions.

1. If
$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \sim N_3(\mu, \Sigma)$$
 where $\mu = \begin{bmatrix} 15 \\ 19 \\ 24 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 19.08 & 7.81 & 6.35 \\ 7.81 & 15.05 & 4.92 \\ 6.35 & 4.92 & 20.52 \end{bmatrix}$

- a) Obtain the marginal distribution of $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$
- b) Compute the multiple correlation coefficient between X_1 & (X_2, X_3) .
- 2. Let $X_1 = Sepal length$

$$X_2$$
 = Petal length

$$X_3 =$$
Sepal width

60 observations were taken from the population Iris varsicolor & 50 observations were taken from the population Iris cotosa. Data summarized is as follows.

$$\overline{X}^{(1)} = (5.94 \ 2.77 \ 1.36)'$$

$$\overline{X}^{(2)} = (5.013.121.46)'$$

$$S = \begin{bmatrix} 0.1773 & 0.0914 & 0.0628 \\ 0.0914 & 0.1107 & 0.0428 \\ 0.0628 & 0.0428 & 0.1131 \end{bmatrix}$$

Assuming the measurements follow the multivariate normal distribution with means $\mu^{(1)}$ and $\mu^{(2)}$ & have the same unknown dispersion matrix Σ .

a) Test the hypothesis $H_0: \mu^{(1)} = \mu^{(2)}$

- b) Find 95% confidenc. Interval for the difference $\mu^{(1)} \mu^{(2)}$ of the population mean vectors.
- 3. Construct a discriminant function using the following data. $n_1 = n_2 = 20$, Means of two groups and the common dispersion matrix are

I II Dispersion matrix
$$X_1 = \begin{bmatrix} 4.74 & 0.56 & 1.47 \\ 0.56 & 0.14 & 0.22 \\ 1.47 & 0.22 & 0.57 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 7.8 & 7.4 \\ X_3 & 10.8 & 10.8 \end{bmatrix}$$

To which group is it possible to assign $X_1 = 27.1 X_2 = 8 X_3 = 11.4$?

4. From data on open/closed book examinations, marks on different topics namely Mechanics(o), Vectors(c), Analysis(o), Statistics(o) [c=closed & o=open], the following matrix of correlation co-efficient was obtained.

$$R = \begin{bmatrix} 302.3 & 125.8 & 106.1 & 116.1 \\ 125.8 & 170.9 & 93.6 & 97.9 \\ 106.1 & 93.6 & 217.9 & 153.8 \\ 116.1 & 97.9 & 153.8 & 294.4 \end{bmatrix}$$

How highly a student's ability on closed book examinations is correlated with his ability on open book examination? Obtain the first canonical vector.