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PGIS-N 1074 B-14

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Statistics

(Linear Algebra)

Paper - HCT - 1.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

Answer any **six** questions from Part-A and any **five** questions from Part-B.

Part - A **(6×5=30)**

1. Find a relationship among the following vectors.
   $X_1 = (5,4,3)$; $X_2 = (4,3,2)$;
   $X_3 = (3,2,1)$ and $X_4 = (1,1,1)$.
2. Show that $k(\geq 2)$ non-null orthogonal vectors are linearly independent.
3. Obtain a basis for the subspace spanned by the vectors X_1 $X_2 X_3$ and X_4 of question 1.
4. Describe Frame's method for finding the inverse of a non singular matrix.
5. Explain Gramschemidt's orthogonalization process.
6. Prove that a square matrix is non singular if and only if it is of full rank.
7. Prove that the characteristic roots of a Hermitian matrix are real
8. Explain the canonical form of a quadratic form.

Part-B **(5×10=50)**

9. a) Prove that $k(\geq 2)$ non-null vectors are linearly dependent if and only if some vector $x_m(2 \leq m \leq k$ is a linear combination of $X_1, X_2, \ldots\ldots X_{m-1}$

   b) Obtain orthonormal vectors using
   $X_1 = (1,-1,1)$; $X_2 = (1,1,-1)$ and $X_3 = (-1,1,1)$ **(5+5)**

10. a) Define
    i) a subspace
    ii) a basis of a subspace
    Prove that a straight line passing through the origin is a subspace

    b) Obtain a basis for the subspace spanned by the vectors of the type $(x_1, x_2, x_3, x_4, x_5)$
    Where $x_1 + x_3 = x_2 + x_4 + x_5$ **(5+5)**

11. a) Prove that the number of members in a basis of a subspace is invariant.

b) Extend (1,1,1) to form a basis for V_3. **(5+5)**

12. a) Obtain a g-inverse of

$$A=\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

b) Find the rank of A by reducing it to its normal form where.

$$A=\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{pmatrix}$$ **(5+5)**

13. a) Define

i) a characteristic root

ii) a characteristic vector.

Prove that every non-null column of $Adj(\lambda I - A)$ gives a characteristic vector corresponding to the characteristic root of A

b) Obtain the characteristic roots and corresponding vectors of

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix}$$ **(5+5)**

14. a) Prove that the characteristic roots of a unitary matrix are of unit modulai.

b) State and prove Caley-Hamilton theorem. **(5+5)**

15. a) Define

i) Algebraic multiplicity (A_m) and

ii) Geometric multiplicity (G_m) of a characteristic root. Prove that GM $\leq$ AM.

b) State and prove sylvester's law of inertia on qundratic forms (q,f). **(6+4)**

16. a) Prove that a q.f. X^1AX is positive difinite if and only if all the eigen values of A are positive

b) Find the canonical form of $x_1^2 + 2x_2^2 + 3x_3^2 - x_1 x_2$ **(6+4)**

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PGIS - N 1082 B - 14

M.A/M.Sc. Ist Semester(CBCS) Degree Examination

Statistics

(Computer Programming in C Language with Statistical Application -I)

Paper : HCP-1.1

(New)

Time : 2 Hours Maximum Marks : 30

Instructions to Candidates

Answer any **two** questions.

1. a) Discuss the basic structure of C Program.

b) Draw a flow chart to find the largest of three numbers.

c) Discuss different types of constants in c-language. **(5+5+5)**

2. a) What are the rules to be followed while choosing avariable name in a C- program?

b) Discuss the advantages of defining a symbolic constants.

c) Which of the following are invalid constants and why?

$897, 14,250 , +5.OE3 , 2E+0.5, E 243. **(5+5+5)**

3. a) What an operator and expression. Discuss Airthmetic and Relational Operators used in C-language.

b) What is meant by casting value ? Explain the outcome of the following Situation

i) X = (Float) 36/5;

ii) P=(int) (a+b) ;

iii) Y = (double) Sum/n;

iv) z=(int) (a+b/c); **(8+7)**

4. a) Discuss Scanf( ) function.

b) Explain 'simple If' statement and 'If------else statement.

c) Write a Program to count the number of boys whose weight is less than 50 kg and height is greater than 170 cm using it Statements. **(5+5+5)**

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PGIS 1085 B-14
M.A/M.Sc. Ist Semester (CBCS) Degree Examination
Statistics
(Based on SCT-1.1)
Paper - SCP-1.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

i) Answer any **four** questions

ii) All questions carry **equal** marks

1. Given the table of fill weights for subgroups of size n=5 calculate UCL and LCL for the $\bar{X}$ and S charts and also construct these charts

Subgroup	X_1	X_2	X_3	X_4	X_5
1	48.85	47.22	45.95	43.04	55.17
2	43.35	54.73	53.82	37.35	43.51
3	53.31	46.34	53.48	63.40	47.51
4	39.83	60.09	54.35	50.53	51.16
5	43.50	48.05	58.97	55.11	48.07

If the specifications are at 705+15 and the process output is normally distributed estimate the fraction nonconforming

2. The counts of complex circuit board assembly errors for 20 subgroups are given below

subgroup:	1	2	3	4	5	6	7	8	9	10	11	12
defect counts:	1	7	3	6	4	2	3	10	3	5	4	1

subgroup	: 13	14	15	16	17	18	19	20
defect count	: 7	2	4	2	4	7	6	8

Construct a chart and comment on the situation. Also draw the o.c. curve for the chart for the following values. 1,2,3,4 ,6,8 and 10

3. Suppose that a single sampling plan with sample size h=150 and allowance number C=2, is being used for receiving inspection where the vendor ships the products in lots of size 3000. Rejected lots are screened and all defectives found are reworked and returned to the lot. Draw O.C, AOQ and ATI curves for this plan

4. A multiple sampling plan is as follows

Sample No	Sample Size	Acceptance No	Rejection No
1	50	-	3
2	50	1	3
3	50	2	4
4	50	3	5
5	50	5	6

Use Poisson distribution to compute probability of acceptance of lot of 3% fraction defective. Assume that the lot size is very large when compared to the sample size

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PGIS N 1076 B-14
M.A./M.Sc. Ist Semester (CBCS) Degree Examination
Statistics
(Probability Theory)
Paper - HCT - 1.2
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

Answer any **six** questions from Part-A and any five questions from Part-B.

Part - A

1. Define $\lim A_n$, $\underline{\lim} A_n$ and $\overline{\lim} A_n$. Find $\lim A_n$ if $A_n = \{w : 0 < w < 1 - \frac{1}{n}\}$ **(6×5=30)**
2. Show that a field is closed under finite unions. Conversely, a class closed under complimentation and finite unions is a field.
3. Define
   i) Simplefunction
   ii) Elementary function &
   iii) Function of a function.
4. Let X be a random variable difined on (Ω, A) and a and b are constante then show that $ax + b$ is also a random. variable.
5. Define expectation of a random variable X. and show that if $X \geq Y$ $a.s \Rightarrow Bx \geq EY$
6. Define
   i) Convergence of a random variable
   ii) Convergence in probability &
   iii) Convergence almost surely (a.s)
7. Define characteristic function. Show that the characteristic function of a gamma distribution with parameters (α, p) is $(\alpha / \alpha - it)^p$

8. State and prove Techebychev's WLLN's.

Part-B **5×10=50**

9. Define fields and σ-fields. Show that intersections of arbitrary number of fields in a field.

10. Define Inverse function. Show that if A is a class of subsets of Ω and is a σ-field, then the class B of all sets whose inverse image belong to A is also a σ-field.

11. If BX, BY and BX+BY exists, then show the following

    i) B (CX) = CBX

    ii) B(X+Y) = BX+BY.

12. State and prove Markov inequalites.

13. Show that if $X_n \xrightarrow{P} O$ if and only if $E\left(\frac{|x_n|}{1+|x_n|}\right) \to O \; as \; n \to \infty$

14. State and prove Patous limma.

15. If $\phi_X(t)$ is the characteristic function of random variable X then show that the characteristic function of a+bx is eiat $\phi_x(bt)$. Also show that $\overline{\phi}(t)$ is the characteristic function of -X and is real if and only if X is symmetric about the origin

16. State and prove Liapounovls form of CLT

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PGIS 1083 B-14

M.A/M.Sc. Ist Semester (CBCS) Degree Examination

Statistics

(Computer programming in C language with statistical applications-II)

Paper - HCP-1.2

Time : 3 Hours Maximum Marks : 30

Instructions to Candidates:

Answer any **Two** full question.

1. a) What is a 'for' statement? Outline its special features

b) Write a program using 'for' statement to compute and print the sum of the fourth poowers of the first n natural numbers **(8+7)**

2. a) Distinguish between one dimensional and two dimensional arrays. Give examples

b) Write a program to compute and print the standard deviation of raw data using an array **(7+8)**

3. a) What are user defined functions? Discuss the format of a user defined function write a user defined function to compute and print the product of these numbers

b) Discuss the declaration and initialization of a structure. **(8+7)**

4. Write short notes on any **three** of the following **(5 each)**

a) While and do statement

b) category of user defined functions

c) Unions

d) Pointers

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PGIS-N 1078 B-14

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Statistics

(Estimation Theory)

Paper - HCT - 1.3

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

Answer any **six** questions from Part-A and any **five** questions from Part-B.

Part - A (6×5=30)

1. Define an unbiased estimator. Let $X_1, X_2 .... X_n$ be a random sample of size on a r.v X having a finite mean μ and a finite variance σ^2. Prove that $E(\bar{x}) = \mu$ & $E(s^2) = \sigma^2$ where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $(n-1)\ s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$

2. Define a consistent estimator of a parametric function. State and prove a set of sufficient conditions for consistency

3. Show that the sample mean is consistent for 'θ' when the random sample is drawn from $f_0(x) = 2\theta^2/(x+\theta)^3$; if $x>0$ & $f_\theta(x) = 0$, otherwise

4. Define sufficiency of an estimator. Let $X \sim U(\alpha, \beta)$. Where both α & β are unknown. Obtain a sufficient statistic for (α, β)

5. What is a minimal sufficient statistic? Describe a method of obtaining such a statistic

6. Define a Real Parameter exponential family (RPEF). Examine whether $\{U(0,\theta), \theta > 0\}$ is a RPEF.

7. Define CAN&BAN estimators. Prove that under regularity conditions MLE is both CAN&BAN.

8. What are UMA and UMAU confidence sets:

Part-B (5×10=50)

9. a) Let T be the sample sum when the random samples of size (≥ 2) is drawn from $b(1,\theta)$. Examine Whether $\frac{T(T-1)}{n(n-1)}$ is unbiased for θ^2

   b) Y_1 is the minimum in a random sample of size $n(\geq 2)$ from $U(0,\theta)$. Find an unbiased estimator of θ^K based on Y_1 where $K(\geq 1)$ is an integer (5+5)

10. a) Let T_n be consistent for $g(\theta)$, Show that $a_n T_n + b_n$ is also consistent for $g(\theta)$, where $\{a_n\}$ & $\{b_n\}$ are sequences of real numbers such that $\lim_{n\to\infty} a_n = 1$ and $\lim_{n\to\infty} b_n = 0$

    b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n on a r.v. x having finite mean μ and finite variance σ^2. Show that $\frac{2}{n(n+1)} \sum_{i=1}^{n} ix_i$ is consistent for μ. (5+5)

11. a) State the factorisation theorem on sufficiency for a discrete case.

    b) Obtain a sufficient statistic for (μ,σ^2) in $N(\mu,\sigma^2)$ (5+5)

12. a) Stating the necessary assumption state and prove Cramer Rao inequality

    b) Examine whether the M.V.B estimator exists in $N(\mu,1)$. If exists, obtain the estimator, corresponding parametric function, the information function and the variance of the estimator. (5+5)

13. a) State and Prove Rao-Blackwell theorem.

    b) Find the UMVE P^N using a random sample of size n>1, from b(N,p) (5+5)

14. a) State and prove Lehman-Scheffe theorem.

    b) Define Information function. With usual notation show that $i(\theta) = V\left(\frac{d}{d\theta} \log f_\theta(x)\right)$ (5+5)

15. a) Define M.L.E. How is it obtained in practice?

    b) Obtain M.L.E. of θ for $U(0,\theta)$ for (5+5)

16. Write short notes on any **two** of the following: (5+5)

    a) Efficiency

    b) Completeness

    c) Method of moments

    d) Shortest length confidence interval

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PGIS - N 1084 B-14

M.A/M.Sc. Ist Semester (CBCS) Degree Examination

Statistics

(Based on HCT 1.1 and HCT 1.3)

Paper - HCP-1.3

(New)

Time : 2 Hours Maximum Marks : 30

Instructions to Candidates:

Answer any two full questions

1. a) Use frames method to find the inverse of the following matrix

$$\begin{bmatrix} 73.96 & 39 & 11.91 \\ 39 & 31.36 & 7.96 \\ 11.91 & 7.96 & 3.24 \end{bmatrix}$$

b) Obtain the G-inverse of the following matrix $\begin{bmatrix} 0.4 & 0.3 & 0.1 & 0 \\ 0.3 & 0.5 & 0.2 & 0 \\ 0.1 & 0.2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(7+8)**

2. If gene frequencies are in equilibrium the genotype AA,Aa and aa occur in a population with frequencies $(1-\theta)^2, 2\theta(1-\theta)$ *and* θ^2 according to Hardy weinberg law ina s sample from the chinese population of hongkong in 1937, blood types occurred with the following frequencies where M and N are erythrocyte antigens

	Blood type			
	M	MN	N	total
frequencies	342	500	187	1029

Find maximum likelihood estimator(MLE) of θ. **(15)**

3. In an experiment the observed frequencies in four classes are 32,102,122 and so respectively according to theory, the probabilities or occurrences are

$(1-P)^3, 3p(1-p)^2, 3p^2(1-p)$ *and* $p^3, 0 < p < 1$. Obtain MLE of p upto 3 decimal places and also obtain an estimate of its standard error. **(15)**

4. a) The following data gives the yield of brown seeds from square cells of 5ftx5ft. construct 95% confidence interval for the mean yield of standard deviation is 9. The data are as follows

120,149,114,110,96,88,98,114,92,94,100,105,79,83,112,99,87,127,132,145

b) For the data in 4a, construct 95% confidence interval for σ^2 if the man yield is 102

(8+7)