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PGIIIS-N 1529 B-15

M.Sc. IIIrd Semester (CBCS) Degree Examination

Mathematics

(Functional Analysis)

Paper -HCT 3.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any **five** questions
2) All questions carry **equal** marks

1. a) Define a metric space. If (X,b) is a metric space and $A \subset X$, then prove the following

i) $\overline{A}$ is a closed set

ii) A is closed if and only if $A = \overline{A}$

iii) $\overline{A}$ is the smallest closed subset of X containing A

iv) $\overline{A}$ is the intersection of all closed subsets of X containing A **(8)**

b) State and prove cantor's intersection theorem **(8)**

2. a) State and prove Heine-Borel theorem **(8)**

b) Prove that a metric space is sequentially compact if and only if it is totally bounded and complete. **(8)**

3. a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space N/M is defined by $\|x + M\| = Inf\left\{\|x + M\| / m \in M\right\}$ then show that N/M is a normed linear space. Further if N is a Banach space, then show that N/M is also a Banach Space. **(8)**

b) If X and Y are normed linear spaces then prove that X is topologically isomorphic to Y if and only if there exists a linear map $T : X \to Y$ which is onto and there exists positive constants m and M such that $m\|x\| \le \|T(x)\| \le M\|x\|, \forall x \in X$ **(8)**

4. a) Prove that a product normed linear space X of X_1 and X_2 is complete if and only if X_1 and X_2 are complete (8)

b) State and prove Banach-Steinhaus theorem (8)

5. a) State and prove Hahn-Banach Lemma for real case

b) State and prove open mapping theorem (8)

6. a) State and prove natural embedding theorem (8)

b) Define a Hilbert space state and prove polarisation identity (8)

7. a) If M is a closed convex set in a Hilbert space H, then show that for every $x_0 \in H$ there exists a unique point $y_0 \in M$ such that $\|x_0 - y_0\| = Inf\left\{\|x_0 - y\| : y \in M\right\}$ (8)

b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then show that the linear subspace M+N is also closed (8)

8. a) If y is a fixed vector in a Hilbert space H and if f_y is a scalar valued function on H defined by $f_y(x) = (x, y)$, $\forall x \in H$ then show that f_y is functional in H*. Prove further that $\|y\| = \|f_y\|$ (8)

b) Define a self adjoint operator on a Hilbert space H. Show that an operator T on Hilbert space H is self adjoint if and only if (Tx,x) is real for all x. (8)

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PGIIIS - O 1536 B - 15

M.Sc. IIIrd Semester (Non-CBCS) Degree Examination

Mathematics

(Computational Numerical Method - I)

Paper - 3.3

(Old)

Time : 3 Hours Maximum Marks : 50

Instructions to Candidates:

1) Answer any **Five** questions.
2) All questions carry **equal** marks.

1. a) Derive the Hermite interpolating polynomial (5)

   b) For the following values of $f(x)$ and $f^1(x)$ (5)

x	-1	0	1
$f(x)$	1	1	3
$f^1(x)$	-5	1	7

   Estimate the values of f (-0.5) and f(0.5) using the Hermite interpolation.

2. a) Obtain the piecewise quadratic interpolating polynomials for the function $f(x)$ defined by the data:

x	-3	-2	-1	1	3	6	7
$f(x)$	369	222	171	165	207	990	1779

   Hence find an approximate value of f (-2.5) and f (6.5). (5)

   b) Using the chebyshev polynomials obtain the least squares approximation of second degree for $f(x)=x^4$ *on* $[-1,1]$ (5)

3. a) Describe the Newton - Raphson method of solving system of non-linear equations.(5)

   b) Perform two iterations of Newton-Raphson method to solve (5)

   $x^2+y^2=4$

   $xy-1=0$

   with $(x_o, y_o)=(2,0)$.

4. a) Describe Graeffe's root squaring method. (5)

b) Find all the roots of $x^3 - 8x^2 + 17x - 10 = 0$ by Graeffe's root squaring method. (5)

5. a) Explain Gauss elimination method of solving system of linear algebraic equations.(5)

b) Solve the following system of equations by Gauss - elimination method.

$$5x - y - 2z = 142$$ (5)

$$x - 3y - z = -30$$

$$2x - y - 3z = 5$$

6. a) Explain SOR method of solving system of linear algebraic equations (5)

b) Solve the system of equations

$$3x + 2y = 4.5$$

$$2x + 3y - z = 5$$

$$-y + 2z = -0.5$$

By SOR method find the optimal relaxation parameter and perform three iterations. (5)

7. a) Describe jordan method of matrix inversion. (5)

b) Find the inverse of the matrix (5)

8. a) Describe power method of obtaining largest eigen value and the corresponding eigen vector of a matrix

$$A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$$ (5)

b) Find the largest eigen value in magnitude and the corresponding eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ by using power method. (5)

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M.A./M.Sc. III Semester (Non-CBCS) Degree Examination

Mathematics

(Functional Analysis)

Paper - 3.1

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) Answer any **Five** questions
2) **All** questions carry **equal** marks

1. a) State and Prove cantor's intersection theorem. **(8)**

   b) State and Prove Heine - Borel theorem **(8)**

2. a) If $B(X,R)=\{f/f:x\to R,\ f \text{ is bounded}\}$ where $x\neq\phi$ and norm of f is defined by $\|f\|=\underset{x\in x}{Sup}\ |f(x)|, \forall f\in B(x,R)$, then show that B(x,R) is a Banach space. **(8)**

   b) If x and y are two normed linear spaces and $T:x\to y$ is an onto mapping, then prove that T^{-1} exists and is continuous if and only if $\exists$ a constant m > 0 such that $m\|x\|\leq\|T(x)\|, \forall x\in x$ **(8)**

3. a) Prove that a product normed linear Space x of x_1 and x_2 is complete if and only if x_1 and x_2 are complete. **(8)**

   b) State and prove Banach - Steinhous theorem. **(8)**

4. a) State and prove Hahn-Banach theorem. **(8)**

   b) State and Prove open mapping theorem **(8)**

5. a) State and prove natural embedding theorem. .(8)

b) Define a Hilbert Space. State and Prove polarization identity. (8)

6. a) If M is a closed convex set in a Hilbert space H, then show that for every $x_o \in H$ there exists a unique vector $y_o \in$ M such that $\|x_o - y_o\| = \inf\{\|x_o - y\| : y \in M\}$ (8)

b) If M is a proper closed linear subspace of a Hilbert Space H, then show that there exists a unique vector Z_0 in H such that $Z_0 \perp M$. (8)

7. a) If y is fixed vector in a Hilbert Space H and if fy is a scalar valued function on H defined by $f_y(x) = (x, y), \forall x \in H$, then show that fy is functional in H*. Prove further that $\|y\| = \|f_y\|$ (8)

b) Define a self adjoint operator on a Hilbert space H. If T is an linear operator on a Hilbert space H then prove that $(Tx, x) = O$ if and only if T = O, $\forall x \in H$. (8)

8. a) Define eigen space of an operator T on a Hilbert Space and Spectral resolution of T. (6)

b) State and Prove the Spectral theorem. (10)

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PGIIIS-N 1529 B-15
M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Functional Analysis)
Paper -HCT 3.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any **five** questions
2) All questions carry **equal** marks

1. a) Define a metric space. If (X,b) is a metric space and $A \subset X$, then prove the following

i) $\overline{A}$ is a closed set

ii) A is closed if and only if $A = \overline{A}$

iii) $\overline{A}$ is the smallest closed subset of X containing A

iv) $\overline{A}$ is the intersection of all closed subsets of X containing A **(8)**

b) State and prove cantor's intersection theorem **(8)**

2. a) State and prove Heine-Borel theorem **(8)**

b) Prove that a metric space is sequentially compact if and only if it is totally bounded and complete. **(8)**

3. a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space N/M is defined by $\|x+M\| = Inf\left\{\|x+M\| / m \in M\right\}$ then show that N/M is a normed linear space. Further if N is a Banach space, then show that N/M is also a Banach Space. **(8)**

b) If X and Y are normed linear spaces then prove that X is topologically isomorphic to Y if and only if there exists a linear map $T : X \to Y$ which is onto and there exists positive constants m and M such that $m\|x\| \le \|T(x)\| \le M\|x\|, \forall x \in X$ **(8)**

4. a) Prove that a product normed linear space X of X_1 and X_2 is complete if and only if X_1 and X_2 are complete (8)

b) State and prove Banach-Steinhaus theorem (8)

5. a) State and prove Hahn-Banach Lemma for real case

b) State and prove open mapping theorem (8)

6. a) State and prove natural embedding theorem (8)

b) Define a Hilbert space state and prove polarisation identity (8)

7. a) If M is a closed convex set in a Hilbert space H, then show that for every $x_0 \in H$ there exists a unique point $y_0 \in M$ such that $\|x_0 - y_0\| = Inf\{\|x_0 - y\| : y \in M\}$ (8)

b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then show that the linear subspace M+N is also closed (8)

8. a) If y is a fixed vector in a Hilbert space H and if f_y is a scalar valued function on H defined by $f_y(x) = (x, y)$, $\forall x \in H$ then show that f_y is functional in H*. Prove further that $\|y\| = \|f_y\|$ (8)

b) Define a self adjoint operator on a Hilbert space H. Show that an operator T on Hilbert space H is self adjoint if and only if (Tx,x) is real for all x. (8)

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PGIIIS-O 1538-A B-15

M.A./M.Sc. IIIrd Semester(Non CBCS) Degree Examination

Mathematics

(Fuzzy sets and Fuzzy systems-I (Elective-A))

Paper - 3.6

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any five questions
2) All questions carry equal marks

1. a) State and prove De-morgan's laws of crisp sets **(6)**

b) Explain the notation of fuzzy set with suitable example **(5)**

c) Define α-cut of fuzzy set and explain with example **(5)**

2. a) Define the convexity of fuzzy set and prove that, a fuzzy setA on $\mathbb{R}$ is convex if and only if $\mu A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu A(x_1), \mu A(x_2))$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$ **(8)**

b) Define standard fuzzy set operations and explain with examples **(8)**

3. a) If A,B are the fuzzy sets defined on universal set X then prove that, the following properties hold for all $\alpha \in [0,1]$ **(8)**

i) $^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$

ii) $^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

b) If $A, B \in F(x)$, then prove the following properties holds for all $\alpha \in [0,1]$ **(8)**

i) $A \subseteq B \text{ iff } {}^{\alpha}A \subseteq {}^{\alpha}B$

ii) $A \subseteq B \text{ iff } {}^{\alpha+}A \subseteq {}^{\alpha+}B$

4. a) State and prove second decomposition theorem (8)

b) If a function $C:[0,1]\rightarrow[0,1]$ satisfy axioms C_α and C_4 of fuzzy complements, then prove that it also satisfy axioms C_1 and C_3 (8)

5. a) If C is a continuous fuzzy complement, then prove that C has a unique equilibrium (8)

b) Write axiomatic skeleton for t-narms. Prove that for all $a,b\in[0,1]$

$$i_{\min}(a,b)\leq i(a,b)\leq \min(a,b)$$ (8)

6. a) If i_w denote the class of yager t-norms defined by

$$i_w(a,b)=1-\min\left[1,\left[(1-a)^w+(1-b)^w\right]^{\frac{1}{w}}\right] w>0$$ .Then prove that $i_{\min}(a,b)\leq i_w(a,b)\leq \min(a,b)$

for all $a,b\in[0,1]$ (8)

b) Write axiomatic definition of t-conorms and write one parameterize class of increasing generator and corresponding class of t-conorms. (8)

7. a) If <i,u,c> is a dual triple that satisfies the law of excluded middle and law of contradiction, then prove that <i,u,c> does not satisfy distributive law (8)

b) Define fuzzy number & write an example and also discuss the special cases of fuzzy numbers (8)

8. a) Define linguistic variables and explain with suitable example (8)

b) Write a detailed note on fuzzy equations (8)

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PGIIIS - O 1538 B - 15

M.A/M.Sc. IIIrd Semester (Non - CBCS) Degree Examination

Mathematics

(Fluid Mechanics - I (Elective A))

Paper - 3.5

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

1) Answer any **five** questions

2) All questions carry **equal** marks

1. a) Explain about Lagrange's and Euler's method in fluid motion. Derive $\frac{d\theta}{dt}=\frac{\partial\theta}{\partial t}+\vec{q}.\nabla\phi$. **(8)**

b) Derive equation of continuity by Lagrange and Euler's method. **(8)**

2. a) Derive the Euler's equation of motion in the form. $\frac{\partial\vec{q}}{\partial t}+\vec{q}.\nabla\vec{q}=-\frac{1}{\rho}\nabla p+\vec{k}$, where the symbols have their usual meanings. **(8)**

b) Derive the vorticity transport equation in the form. $\frac{\partial\vec{w}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{w}=\left(\vec{w}.\nabla\right)\vec{q}$, where $\vec{w}=\nabla\times\vec{q}$ is the vorticity vector. Further show that for a 2-dimensional flow vorticity remains constant during the flow. **(8)**

3. a) State and derive Bernoulli's equation. **(8)**

b) Obtain equations of motion for an impulsive action. **(8)**

4. a) Derive the complex potential to a source, sink and doublet. (8)

b) A velocity field is given by $\vec{q}=\frac{(-iy+jx)}{x^2+y^2}$. Determine whether the flow is irrotational or not. Calculate the circulation round having

i) A square with corners at (1,0), (2,0),(2,1), (1,1).

ii) An unit circle with centre at the origin. (8)

5. a) Find the equation of stream lines of flow due to sources of strength 'm' at A(-a,0) and B(a,0) and a sink of strength '2m' at the origin. (8)

b) Discuss the flow due to a uniform line doublet of strength "μ" per unit length at the origin. Sketch the stream lines and potential lines. (8)

6. a) Find the complex potential over an elliptic cylinder. (8)

b) State and prove Blasius theorem. (8)

7. a) State and prove the Milne-Thompson circle theorem. (8)

b) Find the potential function and stream function for the flow due to a stationary cylinder $|z|=a$ and a uniform flow along the negative x-axis. (8)

8. a) State and prove the uniqueness theorem. (8)

b) State and prove Kalvin's minimum energy theorem. (8)

PGIIIS-O 1537 B - 15

M.A./M.Sc. IIIrd Semester (Non-CBCS) Degree Examination

Mathematics

(Graph Theory - I (Elective A))

Paper - 3.4

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) Answer any **Five** full questions.
2) **All** questions carry **equal** marks.

1. a) Show that the maximum number of edges in a simple graph with n - vertices in $\frac{n(n-1)}{2}$. **(5)**

b) Prove that every simple graph must have atleast two vertices of the same degree. **(5)**

c) Show that an edge e of a connected graph G is a bridge of G if and only if e does not lie on a cycle of G . **(6)**

2. a) Prove that a graph G is bipartite if and only if all its cycles are even. **(8)**

b) Show that a (p,q) graph G is a tree if and only if it is a cyclic and p = q+1. **(8)**

3. a) For every nontrivial tree prove that it contains atleast two end vertices. Also construct cubic graphs with 2n vertices $3 \le n \le 6$ having no cycles. **(8)**

b) Prove that the distance between vertices of a connected graph is metric. Draw all r - regular graph with 8 vertices for each $0 \le r \le 6$. **(8)**

4. a) Show that every tree has either one or two centres. **(6)**

b) For any graph G, prove that $K(G) \le \lambda(G) \le \delta(G)$, Also construct a graph with $K = 2, \lambda = 3, \delta = 4$. **(10)**

5. a) In every network, show that the value of a maximum flow equals the capacity of a minimum cut. (8)

b) Find under what condition the complete bipartite graph Km.n has eulerian graph. Draw:

i) An eulerian graph which is not Hamiltonian.

ii) An eulerian graph which is also Hamiltonian (8)

6. a) In a complete graph with n-vertices, show that there are $\frac{n-1}{2}$ edge disjoint Hamiltonian cycles, if n is odd number ≥ 3. (8)

b) If G is a graph with $P \geq 3$ vertices such that for all non adjacent vertices u and v, $\deg u + \deg v \geq P$. Then prove that G is Hamiltonian. (8)

7. a) Show that a graph G is a line graph of a tree if and only if it is a connected block graph in which each cut vertex is on exactly two blocks. (8)

b) Draw the following graphs :

i) $L(C_5)$

ii) $L(K_2, 3)$

iii) $L_3(C_3)$

iv) $L^2(P_6)$

v) $T(C_3)$

vi) $T(K_1, 3)$

vii) $T(P_4)$

viii) $L^3(P_{10})$ (8)

8. a) Show that every acyclic diagraph G has at least one vertex with zero in degree and atleast one vertex with zero out degree. (8)

b) Prove that a tournament is transitive if and only if it is acyclic. (8)

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PGIIIS-N 1532 B-15
M.Sc. IIIrd Semester(CBCS) Degree Examination
Mathematics
(Fluid Mechanics-I)
Paper - SCT-3.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any **five** questions
2) All questions carry **equal** marks

1. a) Explain about Lagrange and Eulerian method in fluid motion derive

$$\frac{\partial \rho}{\partial t}+\nabla\left(\rho,\vec{q}\right)=0$$ (8)

b) Find the path lines and streak lines for the velocity $\vec{q}=\left(\frac{x}{t},y,0\right)$ (8)

2. a) Derive Lamb's hydrodynamical equation (8)

b) Obtain general equations of motion for an impulsive actions (8)

3. a) Define sources and sinks. Find the complex potential for a doublet (8)

b) Derive the complex potential for the image of a doublet relative to a circle.Define conformal transformation (8)

4. a) Fluid is coming out from a small hole of cross section σ_1 in a tank if the minimum cross section of the stream coming out of the hole is σ_2 then show that $\frac{\sigma_2}{\sigma_1}=\frac{1}{2}$ (8)

b) State and prove Blasius theorem (8)

5. a) Discuss the motion of a circular cylinder moving with velocity U along x-axis in an infinite mass of liquid and at rest at infinity (8)

b) A two dimensional flow field is given by $\varphi = xy$ show that the flow is irrotational Find the velocity potential and stream lines (8)

6. a) Derive the expression for kinetic energy of a liquid flow extending to infinity (8)

b) Show that the velocity potential of sphere is $\phi = \left[Ar^{n} + Br^{-(n+1)} \right] p_n(\mu)$ where $\mu = \cos\theta$ and (r, θ, w) are the spherical coordinates (8)

7. a) Liquid is contained in a rotating elliptic cylinder by making use of elliptic transformation $z = C\cosh$ and show that the stream function of the motion is

$$\varphi = \frac{1}{2} w \left(\frac{a^2 - b^2}{a^2 + b^2} \right) \left(x^2 - y^2 \right)$$ (8)

b) Show that fluid pressure exerts a force $\frac{M\dot{U}}{2}$ opposing the motion (8)

8. a) Find the necessary and sufficient condition that vortex lines may be at right angles to the stream lines (8)

b) Derive vorticity transport equation (8)
