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PGIIIS - N 1529 B - 14
M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Functional Analysis)
Paper - HCT 3.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates: Answer any **five** questions
All questions Carry **equal** Marks

1. a) State and prove Cantor's intersection theorem **8**

   b) State and prove Baire's Category theorem. **8**

2. a) Prove that a metric space is sequentially compact if and only if it has the bolzano weierstrass property. **8**

   b) State and prove Lebergue covering lemma **8**

3. a) If $B(X,R)=\{f:x\to R/f \text{ is bounded}\}$, Where $x\neq\phi$ and norm offis defined by

   $\|f\|=\underset{x\in X}{Sup}|f(x)|,\forall f\in B(X,R)$, then Show that $B(X,R)$ is a Banch space **8**

   b) If N and N^1 are normed linear spaces and T is a linear transformation of N into N^1, then prove that the following conditions are equivalent to one another

   i) T is Continuous

   ii) T is Continuous at origin i.e., $x_n\to 0\Rightarrow T(x_n)\to 0$

   iii) $\exists$ a real number $K\geq 0$ such that $\|Tx\|\leq K\|x\|,\forall x\in N$

   iv) If $S=\{x:\|x\|\leq 1\}$ is the closed unit sphere in N, then its image T(S) is bounded in N^1 **8**

4. a) if X_1, and X_2 are two normed linear spaces of same dimension with the scalar filed C or R, then show that X_1 is Topologically isomorphic to X_2 **8**

b) State and prove Riesz's lemma 8

5. a) State and prove Hahn - Banach theorem. 8

b) If M is a Closed linear Subspace of a normed linear space N and x_0 is a vector not in M, then show that there exists a functional f_0 in N^* Such that $f_0(x) = 0, \forall x \in M$,

$f_0(x_0) = 1 \, and \quad \|f_0\| = \frac{1}{d}$ 8

6. a) State and prove closed graph theorem 8

b) Define a Hilbert space, state and prove cauchy - Schwartz's inequality 8

7. a) Prove that a Banch space is a Hilbert space if and only if the parallelgram law holds 8

b) If M is a proper closed subspace of a Hilbert space H, then show that there exists a nonzero vector z_0 in H such that $Z_0 \perp M$ 8

8. a) State and prove Riesz representation theorem. 8

b) Define a self adjoint operation on a Hilbert space H. if T_1 and T_2 are self adjoint operations, then show that T_1T_2 is self adjoint if and only if $T_1T_2 = T_2T_1$ 8

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PGIIIS-N 1532 B-14

M.A/M.Sc. IIIrd Semester (CBCS) Degree Examination

Mathematics

(Fluid Mechanics -I)

Paper - SCT-3.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

1. *Answer any **five** questions*
2. *All questions carry **equal** marks.*

1. a) Derive the condition for a surface to be a possible form of boundary surface **(8)**

b) Define vorticity vector, and velocity potential. Show that surfaces exist which cut stream lines orthogonally if the velocity potential exists **(8)**

2. a) If velocity distribution is $\bar{q} = i(Ax^2yt) + j(By^2zt) + k(Zt^2)$ where A,B,C are constants, then find acceleration and vorticity components **(8)**

b) State and prove Bernoullis equations **(8)**

3. a) Define a doublet find the complex potential for a two-dimensional surface of strength m placed at the origin **(8)**

b) The particle velocity for a fluid motion referred to rectangular axis is given by the components

$$u = A\cos\frac{\pi x}{2a}\cos\frac{\pi z}{2a}$$

$$V = 0,\ W = A\ Sin\frac{\pi x}{2a}\sin\frac{\pi z}{2a}$$

Where A is constant show that this is a possible motion of an incompressible fluid under no body forces in an infinite fixed rigid tube $-a \le x \le a,\ 0 \le z \le 2a$ Also find the pressure associated with this velocity field **(8)**

4. a) Determine wheather the flow is irrotational calculate the circulation round a

i) square with corners at (1,0),(2,0),(2,1) and (1,1)

ii) A unit circle with centre at the origin (8)

b) Prove that for liquid circulating irrotationally in part of the plane between two non-interacting circles the curves of constant velocity are Cassini circles (8)

5. a) Find the velocity potential and stream function in case of an elliptic cylinder rotating in an infinite mass of liquid at rest at infinity (8)

b) Determine the stream function and velocity potential when an elliptic cylinder moves in an infinite liquid with velocity is parallel to major axis of the cross section (8)

6. a) State and prove kulta zoukowsk's theorem (8)

b) A circular cylinder is moving in a liquid at rest at infinity: Calculate the forces on the cylinder owing to the pressure of the liquid (8)

7. a) Determine the lines of flow (stream lines) when a sphere of radius a is moving with velocity W in a liquid at rest at infinity (8)

b) Show that fluid pressure exerts a force $\frac{\dot{M}\dot{U}}{2}$ opposing the motion (8)

8. a) Prove that the product of cross section and the vorticity at any point on a vortex filament is constant along the filament (8)

b) State and prove eulers momentum theorem (8)

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PGIIIS-N 1530 B- 14
M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Graph Theory - I)
Paper : HCT. 3.2
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

i) *Answer any **Five** full questions.*

ii) *All questions carry equal marks.*

1. a) Prove that every u - v walk contains a u - v path. (5)

   b) Prove that an edge 'e' of a connected graph G is a bridge of G if and only if 'e' does not lie on cycle of G. (6)

   c) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw the two such graphs. (5)

2. a) Construct cubic graphs with 6 and 8 vertices. (5)

   b) Show that for any graph G with six vertices G or $\overline{G}$ contains a cycle. Also illustrate through an example. (6)

   c) Prove that every self - complementary graph has 4n or 4n +1 vertices. (5)

3. a) Define a tree. Show that *a (p, q)* graph G is a tree if and only if G is connected & $p = q + 1$ (6)

   b) Show that every nontrivial tree contains atleast two end vertices. (5)

   c) Prove that a connected graph G is a tree if and only if every edge is a bridge. (5)

4. a) Explain the meaning of eccentricity, radius and diameter of a graph through a graph. And show that for every connected graph G,

   $rad\,(G) \leq diam\,(G) \leq 2\,rad\,(G)$ (8)

   b) Prove that in every network, the value of a maximum flow equals the capacity of a minimum cut. (8)

5. a) For any graph G, show that $K(G) \leq \lambda(G) \leq \delta(G)$. Also construct a graph with

$K = 2, \lambda = 3, \delta = 4$. (8)

b) Let G be a nontrivial connected graph. Then prove that G contains an eulerian trail if and only if G has exactly two odd vertices. (8)

6. a) For every nontrivial connected eulerian graph G, show that the set of edges of G can be partitioned into cycles. Also find for what values of 'n' does K_n, a complete graph with n-vertices have an euler circuit. (10)

b) Find under what conditions the complete bipartite graph $K_{m,n}$ has an eulerian circuit. (6)

7. a) If G is a graph with $p \geq 3$ vertices such that for all nonadjacent vertices u and v, deg u + deg $v \geq p$, then prove that G is Hamiltonian. (8)

b) Show that every tournament has a spanning path. (8)

8. a) Define the following digraphs with an example:

i) Simple digraph

ii) Asymmetric digraph

iii) Symmetric digraph

iv) Complete digraph. (6)

b) Prove that every tournament has a Hamiltonian path. (6)

c) Draw all the tournaments with four vertices. (4)

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PGIIIS - N 1531 B - 14

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination

Mathematics

(Computational Numerical Methods - I)

Paper - HCT 3.3

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates: i) Answer any five questions

ii) All questions Carry equal Marks

1. a) Derive the Hermite interpolating polynomial **(8)**

b) For the following values of f(x) and $f^1(x)$

x	-1	0	1
f(x)	1	1	3
$f^1(x)$	-5	1	7

(8)

Estimate the values of f(-0.5) and f(0.5) using the Hermite interpolation

2. a) What is picewise interpolation? Obtain the picewise interpolating polynomials for the function f(x) defined by the data.

x:	1	2	4	8
f(x):	3	7	21	73

Hence estimate the values of f(3) and f(7) **(8)**

b) The following data represents a function f(x,y)

y/x	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

Obtain the bivariate interpolating polynomial which fits this data and hence find the value of f(0.5,0.5) **(8)**

3. a) Explain the Birge-Vieta method for finding the root of a polynomial equation of degree n, $P_n(x)=0$ (8)

b) Perform two iterations of Newton - Raphson method to solve

$$x^2 + y^2 = 4$$

$$xy - 1 = 0 \text{ With } (x_0, y_0) = (2,0)$$ (8)

4. a) Describe Graeffe's root squaring method (8)

b) Find all roots of $x^3 - 8x^2 + 17x - 10 = 0$ by Graeffe's root squaring method (8)

5. a) Describe Gauss -Jordan elimination method for solving system of algebraic equations (8)

b) Solve the following system of equation by LU - decomposition method

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12$$ (8)

6. a) Describe Jacobi's iteration method for solving the system of n-linear equations with n-unknowns (8)

b) Solve the following system of equations by using Jacobi's iteration method

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Take the initial approximation as $\overset{(0)}{x} = [0.5, -0.5, -0.5]^T$ and perform three iterations (8)

7. a) Describe triangularization method for matrix inversion (8)

b) Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$ by triangularization method (8)

8. a) Describe Givan's method to find eigen values and the corresponding eigen vectors of a real symmetrix matrix (8)

b) Using Jacobi's find all the eigen values and corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{3} & 3 & \sqrt{2} \\ 2 & \sqrt{3} & 1 \end{bmatrix}$$ (8)

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PGIIIS - N 1533 B - 14
M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Operations Research - II)
Paper : OET 3.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

i) Answer any **five** questions

ii) All questions carry **equal** Marks.

1. a) Explain two phase method of Solving L.P.P. **(8)**

b) Solve the following L.P.P by two phase method **(8)**

$$\begin{aligned} &\textit{Minimize } z = x_1 + 2x_2 + 3x_3 \\ &\textit{Subject to } x_1 - x_2 + x_3 \geq 4 \\ &\qquad x_1 + x_2 + 2x_3 \leq 8 \\ &\qquad x_1 - x_3 \geq 2 \\ &\textit{and } x_1, x_2, x_3 \geq 0 \end{aligned}$$

2. a) Write the dual of the following L.P.P. **(8)**

$$\begin{aligned} &\textit{Minimize } z = 2x_2 + 5x_3 \\ &\textit{Subject to } x_1 + x_2 \geq 2 \\ &\qquad 2x_1 + x_2 + 6x_3 \leq 6 \\ &\qquad x_1 - x_2 + 3x_3 = 4 \\ &\textit{and } x_1, x_2, x_3 \geq 0 \end{aligned}$$

b) Solve the following L.P.P by dual Simplex method **(8)**

$$\begin{aligned} &\textit{Minimize } z = 5x_1 + 6x_2 \\ &\textit{Subject to } x_1 + x_2 \geq 2 \\ &\qquad 4x_1 + x_2 \geq 4 \\ &\textit{and } x_1, x_2 \geq 0 \end{aligned}$$

3. a) Explain (8)
i) Two - Person zero - sum games.
ii) Payoff - matrix.

b) Solve the following game and determine the value of the game (8)

Player B

$$\text{Player A} \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

4. a) Write the graphical method for $2\times N$ or $M\times 2$ games. (8)

b) Using the Principle of dominance, solve the following game. (8)

Player B

$$\text{Player A} \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

5. a) Explain arrival process. (8)

b) A xerox machine in an office is operated by a person who does other jobs also. The average service time for a job is 6 minutes per customer -on an average, every 12 minutes one customer arrives for xeroxing. Find (8)
i) The xerox machine utilization
ii) Percentage of times that an arrival has not to wait
iii) Average time spent by a customer
iv) Average queue length.

6. a) Discuss (M/M/l) : $(GD/\infty/\infty)$ model. (8)

b) In a car wash Service facility information gathered indicates that cars arrive for service to a Poisson distribution with mean 5 per hour. The time for washing and cleaning each car varies but is found to follow an exponential distribution with mean 10 minutes per car. The facility can not handle more than one car at a time. Find expected number of cars in queue and expected waiting time in System. (8)

7. a) Illustrate how the Random observations can be Computed for
i) Erlang distribution
ii) Normal distribution
iii) Poisson distribution. (8)

b) Discuss Simulation languages. (8)

8. a) Discuss the advantages and limitations of Simulation models (8)

b) Discuss the Monte-Carlo simulation technique (8)