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PGIIS 1009 A-16
M.A./M.Sc. IInd Semester(CBCS) Degree Examination
MATHEMATICS
(Operations Research-I)
Paper : OET-2.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) *Answer any **five** questions*
2) *All questions carry **equal** marks*

1. a) Define Operation Research (OR). What is its Scope? Explain briefly. **(8)**

b) A Company has three operational departments (Weaving, Processing and Packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs 2, Rs 4 and Rs 3 per metre respectively. One metre of suiting requires 3 minutes in weaving, 2 minutes in Processing and 1 minute in packing. Similarly one metre of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One metre of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively. Formulate the linear programming problem to find the product mix to maximize the profit. **(8)**

2. a) Solve the following LPP by graphical method.

Maximize $Z = 3x_1 + 10x_2$

Subject to $3x_1 + 4x_2 \leq 24$

$-x_1 + 5x_2 \leq 15$

and $x_1, x_2 \geq 0$ **(8)**

b) Solve the following LPP by simplex method.

Maximize $Z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

and $x_1, x_2 \geq 0$ **(8)**

3. a) Use Big-M method to solve the following LPP

Maximize $Z = x_1+2x_2+3x_3-x_4$

Subject to $x_1+2x_2+3x_3 = 15$

$2x_1+x_2+5x_3 = 20$

$x_1+2x_2+x_3+x_4=10$

and $x_1, x_2, x_3, x_4 \geq 0$ (8)

b) Explain the method of column minima and matrix minima to obtain initial basic feasible solution to the given transportation problem. (8)

4. a) Explain the method of row minima to obtain initial basic feasible solution to the given transportation problem. (4)

b) Determine the initial basic feasible solution to the following transportation problem by using

i) North-West Corner rule

ii) Vogel's Approximation Method (VAM)

(Destinations)

(Origins)		A	B	C	D	(Availability)
	I	21	16	25	13	11
	II	17	18	14	23	13
	III	32	27	18	41	19
(Requirements)		6	10	12	15	

(12)

5. a) Write the mathematical formulation of the assignment problem. (8)

b) Solve the following assignment problem (8)

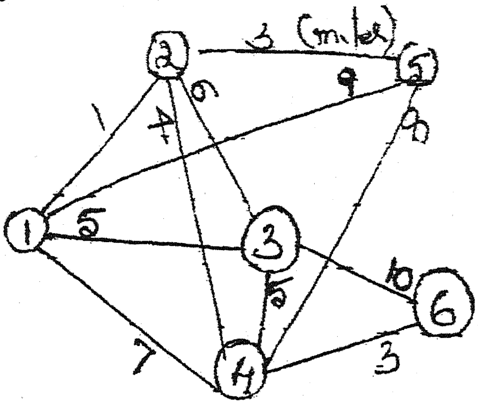
	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	15

6. a) Solve the following travelling salesman problem so as to minimize the cost per cycle

		To				
		A	B	C	D	E
From	A	-	3	6	2	3
	B	3	-	5	2	3
	C	6	5	-	6	4
	D	2	2	6	-	6
	E	3	3	4	6	-

(8)

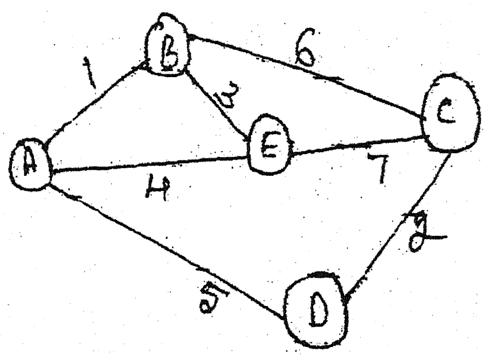
b) The Midwest T.V. cable company is in the process of providing cable services to five new housing development areas. The figure below depicts the potential T.V. linkages among the five areas. The cable miles are shown on each branch.



Determine the most economical cable network for the Midwest company. (8)

7. a) Write the Dijkstra's shortest path algorithm. (6)

b) Use Dijkstra's algorithm to determine a shortest path from A to C for the following network : (10)



8. a) Write fractional cut method-all integer LPP. (6)

b) Find the optimum integer solution to the following LPP :

Maximize $Z = x_1 + 4x_2$

Subject to $2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$ and are integers. (10)

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PGIIS 1008 A-16
M.A/M.Sc. IInd Semester(CBCS) Degree Examination
MATHEMATICS
(Complex Analysis)
Paper : SCT-2.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

1) Answer any **FIVE** questions
2) All questions carry equal marks

1. a) Prove that the analytic function f(z) with constant modulus is constant. **(6)**

b) State and prove Cauchy's Theorem for a rectangle. **(10)**

2. a) Evaluate $\int_c \frac{dz}{(z^2+r^2)}$ where 'c' is a simple closed curve not passing through the point $z = \pm r$ **(6)**

b) State and prove Maximum Modulus Theorem. **(10)**

3. a) If $\sum_{n=0}^{\infty} Z_n$ and $\sum_{n=0}^{\infty} w_n$ be two absolutely convergent series. Let $C_n = \sum_{k=0}^{n} z_k w_{n-k}$. Show that $\sum_{n=0}^{\infty} C_n$ is absolutely convergent and $\sum_{n=0}^{\infty} C_n = \left(\sum_{n=0}^{\infty} z_n\right)\left(\sum_{n=0}^{\infty} w_n\right)$. **(6)**

b) State and prove weierstrass-M-Test. **(6)**

c) Determine the radius of convergence for the power series $\Sigma \frac{Z^n}{n!}$ **(4)**

4. a) Let f be analytic in Ω then show that f can be represented by a power series $f(z)=\sum_{n=0}^{\infty} a_n (z-a)^n$ about each point $a \in z$. **(8)**

b) Show that the coefficients in the Laurent expansion of $f(z) = \sin\left(z+\frac{1}{z}\right)$ in powers of z. are given by the formula. $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos n\theta . \sin(2\cos\theta) d\theta$ n=1, 2 -------- **(8)**

5. a) Show that the zero's of an analytic function are isolated. (8)

b) Determine the singularities in each of the following functions. Find the singular part if it is a pole. Define f(0) if it is a removable singularity so that 'f' is analytic at z=0.

i) $f(z) = \dfrac{z^2}{z(z-1)}$

ii) $f(z) = z^2 \sin\left(\dfrac{1}{z}\right)$ (8)

6. a) Evaluate $\int_0^{\infty} \dfrac{2x^2 - 1}{x^4 - 5x^2 + 4} dx$ (8)

b) Show that $\int_0^{2\pi} \dfrac{d\theta}{1 + a\sin\theta} = \dfrac{2\pi}{\sqrt{1-a^2}} \; (-1 \le a \le 1)$ (8)

7. a) State and prove Schwartz lemma (6)

b) State and prove Riemann mapping Theorem. (10)

8. a) Let f be a monomorphic in Ω with poles s_1, s_2, ------s_m and zeros z_1, z_2, ----z_n. If r is closed chain in Ω Homologus to zero in Ω and not passing through s_1, s_2, ------s_m; z_1, z_2, ----z_n then show that $\dfrac{1}{2\pi i}\int_r \dfrac{f^1}{f} = \sum_{k=1}^{n} n(r, z_k) - \sum_{j=1}^{m} n(r, s_j)$. (8)

b) State and prove weirstrass factorization Theorem. (8)

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PGIIS 1005 A-16

M.A/M.Sc IInd Semester(CBCS) Degree Examination

Mathematics

(Partial Differential Equations)

Paper : HCT-2.1

Time : 3 Hours Maximum Marks : 80

Instructions to candidates :

1) Solve any FIVE questions

2) All questions carry equal marks.

1. a) Define the following for a partial differential equation

i) Quasilinear equation

ii) Semilinear equation

iii) Linear equation

iv) Non-Linear equation with examples. **(8)**

b) Find the integral surface of the partial differential equation $y(x+xz)p+(x+yz)q=z^2-1$, which passes through the parabola $x=t, y=1, z=t^2$. **(8)**

2. a) If a characteristic strip contains at least one integral element of $F(x, y, z, p, q)=0$ then prove that it is an integral strip of the equation $F(x, y, z, p, q)=0$. **(8)**

b) Find the solution of the equation $Z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through the x-axis **(8)**

3. a) Write short note on surfaces orthogonal to a given system of surfaces. **(6)**

b) Find the surface which intersect the surfaces of the system $z(x+y) = c\,(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1$. **(10)**

4. a) Write short note on higher - order equations in physics. **(8)**

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{d^2 z}{\partial y^2}$ to canonical form. **(8)**

5. a) Derive the necessary condition for the families of equipotential surfaces. **(8)**

b) Show that the surfaces $x^2+y^2+z^2 = c.x^{2/3}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function. (8)

6. a) Solve the heat equation $\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$ using variable separable method. (8)

b) A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity in the undisturbed state is V_0.Determine the velocity of the fluid at any point of the disturbed stream. (8)

7. a) Explain elementary solution of wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$. (8)

b) Explain the method of obtaining solution of one dimensional wave equation. (8)

8. Solve the following nonlinear partial differential equations.

a) $r+4s+t+rt-s^2 = 2$ (8)

b) $q^2r-2pqs+p^2t = 0$. (8)

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PGIIS 1007 A-16
M.A/M.Sc. IInd Semester(CBCS) Degree Examination
MATHEMATICS
(Programming in C)
Paper : HCT-2.3

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

1) Answer any FIVE questions
2) All questions carry equal marks.

1. a) Explain the functions of various units of a computer with the help of a neat block diagram **(8)**

b) Discuss the evolution of computers giving the details of various generations. **(8)**

2. a) Discuss the basic structure of C-program. **(8)**

b) Define constants and variables. Explain the rules for forming constants and variables. **(8)**

3. a) What is data type? Discuss the four basic data types in C. **(8)**

b) Explain data type modifiers. **(8)**

4. a) Explain formatted and unformatted output statements. **(8)**

b) Write a C - program that accepts the temperature in Fahrenheit and Converts it into Celsius. **(8)**

5. a) Discuss logical and relational operators in C. **(8)**

b) Discuss increment and decrement operators. **(8)**

6. a) What is ternary operator? Explain with an example. **(8)**

b) List special operators in C? Explain sizeof ( ) operator with an example. **(8)**

7. a) Discuss if-else and switch statements with an example. **(8)**

b) Write a C-program to find the largest among the three input integer numbers. **(8)**

8. a) List the different types of loop control statements and explain any one with a suitable example. **(8)**

b) Explain continue statement with a suitable example. **(8)**

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PGIIS - 1006 A - 16
M.A./M.Sc. IInd Semester(CBCS) Degree Examination
Mathematics
(Algebra - II)
Paper : HCT 2.2

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

1. Answer any **Five** questions
2. All questions carry **equal** marks.

1. a) If $v_1, v_2, ---, v_n$ is a basis of V over F and if $w_1, w_2, ---, w_m$ in V are linearly independent over F, then prove that $m \leq n$. **(8)**

b) If A is an algebra with unit element over F, then show that A is isomorphic to a subalgebra of A(v) for some vector space V over F. **(8)**

2. a) Show that if V is an n-dimensional vector space over F, then given any element T in A(V), there exists a nontrivial polynomial $q(x) \in F[x]$ of degree at most n^2, such that $q(T)=0$ **(8)**

b) For a finite dimensional vector space V over F, prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is Nonzero **(8)**

3. a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then show that for any polynomial $q(x) \in F[x]$. $q(\lambda)$ is a characteristic root of q(T) **(8)**

b) If $T, S \in A(V)$ and if S is regular, prove that T and $ST S^{-1}$ have the same minimal polynomial. **(8)**

4. a) If $\lambda_1, \lambda_2, ---- \lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and prove that if v_1, v_2,-----, v_k are characteristic vectors of T belonging to $\lambda_1, \lambda_2, ---- \lambda_k$, then $v_1, v_2, ---- v_k$ are linearly independent over F. **(8)**

b) Show that if $W \subset A$ is invariant under T, then T induces a linear transformation $\bar{T}$ on V/W defined by $\bar{T}(v+W)=Tv+W$. If T satisfies the polynomial $q(x) \in F[x]$, then so does $\bar{T}$. If $P_1(x)$ is the minimal polynomial for $\bar{T}$ over F and if p(x) is that for T, then $p_1(x)/p(x)$ **(8)**

5. a) Prove that if V is n-dimensional over F and if $T \in A(V)$ has all its characteristics roots in F. Then T satisfies a polynomial of degree n over F. (8)

b) Let $T \in A(V)$ and for $v \in V$, $T^k(v)=0$ and $T^{k-1}(v) \neq 0$, then prove the following

a) The set $S=\{v, Tv, T^2v, ---- T^{k-1}v\}$ is linearly independent.

b) The subspace W generated by S is T-invariant. (8)

6. a) If $T \in A(V)$ is such that $<Tv, v>=0$ for all $v \in V$, then prove that T=0 (8)

b) Prove that the linear transformation T on V is unitary if and only if it takes orthonormal basis of V into an orthonormal basis of V. (8)

7. a) State and prove fundamental theorem of arithmetic (6)

b) Find the positive integer solution of the Diophantine equation 7x+19y=213 (5)

c) Show that for arbitrary integers a and b, $a \equiv b \pmod{m}$ if and only if a and b leave the same nonnegative reminder when divided by m. (5)

8. a) Let p be an odd prime and gcd (a,p)=1, Then prove that a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$ (6)

b) For each positive integer $n \geq 1$, Show that , $n = \sum_{d/n} \phi(d)$ the sum being extended over all positive divisors of n. (5)

c) Prove that the function μ is a multiplicative function. (5)