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PGIS-N 1027 B-18

M.Sc. I Semester Degree Examination

MATHEMATICS

(Operations Research)

Paper - SCT 1.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any FIVE questions.*
2. *All questions carry **equal** marks.*

(5×16=80)

1. a. Discuss scientific methods in O.R. **(8)**

   b. A city hospital has the following minimal daily requirements for nurses:

Period	Colck time (24 hours day)	Minimal number of nurses required
1	6 A.M - 10 A.M	2
2	10 A.M. - 2 P.M.	7
3	2 P.M.-6 P.M.	15
4	6 P.M. -10 P.M.	8
5	10 P.M. - 2 A.M.	20
6	2 A.M.- 6 A.M.	6

Nurses report to the hospital at the begining of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate the problem as a linear programming problem. **(8)**

2. a. Solve the following L. P. P. by graphical method.

Maximize $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \le 30$

$x_2 \ge 3$

$0 \le x_2 \le 12$

$0 \le x_1 \le 20$

$x_1 - x_2 \ge 0$

and $x_1, x_2 \ge 0$ **(8)**

b. Solve the following L. P. P. by simplex method

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$ **(8)**

3. a. Use Big-M method to solve the following L. P. P.:

Minimize $z = 5x_1 + 3x_2$

Subject to $2x_1 + 4x_2 \le 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \ge 10$

and $x_1, x_2 \ge 0$ **(8)**

b. Use two-phase simplex method to solve the following L. P. P.

Maximize $z = 5x_1 - 4x_2 + 3x_3$

Subject to $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$

and $x_1, x_2, x_3 \geq 0$ **(8)**

4. a. Write the dual of the following L. P. P.

Maximize $z = 5x_1 + 3x_2$

Subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

and $x_1, x_2 \geq 0$ **(6)**

b. Solve the following L.P.P. by dual simplex method

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

and $x_1, x_2 \geq 0$ **(10)**

5. a. Explain the method of row minima and matrix minima to obtain initial basic feasible solution to the given transportation problem. **(10)**

b. Determine the initial basic feasible solution to the following transportation problem by using North- West corner rule.

(Destinations)

		A	B	C	(Availability)
(Origins)	I	50	30	220	1
	II	90	45	170	3
	III	250	200	50	4
	(Requirements)	4	2	2	

(6)

6. a. Write the transportation technique. **(4)**

b. Find the optimum transportation schedule for the following problem

(Destinations)

		D_1	D_2	D_3	D_4	
(Source)	0_1	10	0	20	11	15
	0_2	12	7	9	20	25
	0_3	0	14	16	18	5
		5	15	15	10	

(Supply)

(Demand) **(12)**

7. a. Explain the Hungarian method of assignment. **(8)**

b. Solve the following assignment problem

	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

(8)

8. a. Determine which of the following two- person zero-sum games are strictly determinable and fair. Give the optimum strategies for each player in the case of strictly determinable games.

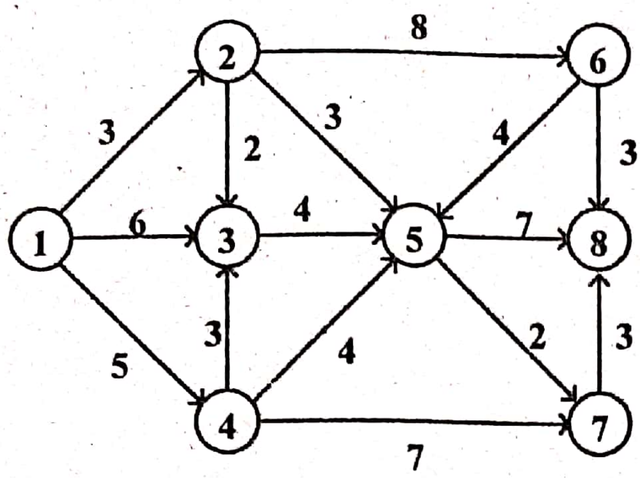
Player B

i. Player A $\begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix}$

Player B

ii. Player A $\begin{bmatrix} 10 & 6 \\ 8 & 2 \end{bmatrix}$ **(8)**

b. Consider the following network.



The distance (in miles) between different stations is shown on each link. Determine the shortest route from station 1 to station 8. **(8)**

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PGIS-O 1026 B-18

M.A./M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

(Classical Mechanics)

Paper - SCT 1.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any **FIVE** full questions.*
2. *All questions carry **equal** marks.*

1. a. State and prove D' Alemberts principle. **(8)**

   b. Derive Lagrange's equation for impulsive motion. **(8)**

2. a. Construct Lagragian and the equations of motion of a coplanar double pendulum placed in an uniform gravitational field. **(8)**

   b. Derive Euler's dynamical equations of motion of a rigid body about a fixed point. **(8)**

3. a. Establish Euler's dynamical equation of motion of a symmetrical top. **(8)**

   b. Deduce Lagrange equations from Hamilton's principle. **(8)**

4. a. State Lee Haw - Chungs theorem and deduce Poincare integral invariant. **(8)**

   b. Establish Hamilton- Jacobi's equation. **(8)**

5. a. Obtain H-J equations for simple harmonic motion and find a complete integral and determine solution of it. **(8)**

   b. By using Poisson's bracket establish Jacobi's identity. **(8)**

6. a. Prove that Poisson's bracket of two constants of motion is itself a constant of a motion. **(8)**

   b. Show that Poisson's bracket are also invariant under canonical transformation. **(8)**

(2)

7. a. Find p and q for a harmonic oscillator described by the Hamilton $H = \frac{1}{2}\left(p^2 + w^2 q^2\right)$ and generated by $F = \frac{1}{2} w q^2 \cos 2\pi Q$. **(8)**

b. A particle of mass m is falling under gravity. Solve for the motion of the particle using canonical transformation. **(8)**

8. a. Prove that Lagrange's bracket is invariant under canonical transformation. **(8)**

b. Find fundamental Lagrange's bracket. **(8)**

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PGIS-O 1019 B-18

M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

(Real Analysis)

Paper - HCT 1.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any **FIVE** questions.*
2. *All questions carry **equal** marks.*

(5×16=80)

1. a. Define Riemann- Steiltje's integrals and describe their existance. Prove that f is integrable with respect to α over [a, b] if and only if for every $\varepsilon > 0$ and for every partition P of [a, b] such that

$U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon.$ **(8)**

b. If $f \in \mathbb{R}(\alpha_1)$ and $f \in \mathbb{R}(\alpha_2)$ then show that $f \in \mathbb{R}(\alpha_1 + \alpha_2)$ and $\int_a^b fd(\alpha_1+\alpha_2) = \int_a^b fd\alpha_1 + \int_a^b fd\alpha_2$ and if $f \in \mathbb{R}(\alpha)$ and c is a positive constant then prove that $f \in \mathbb{R}(c\alpha)$ and $\int_a^b fd(c\alpha) = c\int_a^b fd\alpha$ **(8)**

2. a. If f is monotonic on [a, b] and α is continuous on [a, b] then show that $f \in \mathbb{R}(\alpha)$. **(8)**

b. State and prove first mean value theorem. **(8)**

3. a. Define the meaning of functions of bounded variation. Show that a bounded monotonic function is a function of bounded variation. **(8)**

b. State and prove the fundamental theorem of calculus. **(8)**

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4. a. If $\{f_n\}$ be a sequence of functions such that $\lim_{n \to \infty} f_n(x) = f(x), x \in [a,b]$ and let $M_n = Sup|f_n(x) - f(x)|, x \in [a,b]$ then prove that $f_n \to f$ uniformly on [a, b] if and only if $M_n \to 0$ as $n \to \infty$. **(8)**

b. Show that there exists a continuous function on the real line which is nowhere differentiable. **(8)**

5. a. If f and g are complex functions of bounded variation on[a, b] and f is continuous on [a, b] then prove that $\int_a^b f d\alpha = f(b)\alpha(b) - f(\alpha)\alpha(a) - \int_a^b \alpha df$ **(8)**

b. Prove that if f' exists and is bounded on [a, b], then f is of bounded variation. **(8)**

6. State and prove Stone - Weierstrass theorem. **(16)**

7. a. Prove that a linear operation T on a finite dimensional vector space X is one if and only if the range of T is all of X. **(8)**

b. If X is a complete metric space and if ϕ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\phi(x) = x$. **(8)**

8. State and prove inverse function theorem. **(16)**

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PGIS-N 1028 B-18
M.Sc. I Semester Degree Examination
MATHEMATICS
(Fuzzy Sets & Fuzzy systems)
Paper - SCT 1.2
(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

i. *Answer any **five** questions.*

ii. *All questions carry **equal** marks.*

(5×16=80)

1. a. Define a Fuzzy set and distinguish between crisp set and Fuzzy set. **(4)**

   b. Define the following. **(6)**

      i. Support of a fuzzy set

      ii. α-cut of a fuzzy set

      iii. Level set of a fuzzy set.

      Explain with suitable examples.

   c. Prove that a fuzzy set A on $\mathbb{R}$ is convex if and only if

      $\mu_A(\lambda x_1+(1-\lambda)x_2)\geq \min[\mu_A(x_1),\mu_A(x_2)]$ for all $x_1,x_2\in R$ and all $\lambda\in[0,1]$. **(6)**

2. a. Define standard fuzzy operations and explain with suitable examples. **(8)**

   b. Show that, the Demorgans laws are satisfied for the tree pairs of fuzzy sets A, B and C

      with $\mu_A(x)=\dfrac{1}{1+20x}$; $\mu_B(x)=\left(\dfrac{1}{1+10x}\right)^{1/2}$; $\mu_C(x)=\left(\dfrac{1}{1+10x}\right)^{2}$. **(8)**

3. a. Let $A,B\in F(X)$. Then prove that for all $\alpha\in[0,1]$ the following properties hold.

      i. A=B iff $\alpha_A=\alpha_B$

      ii. A=B iff $\alpha+_A=\alpha+_B$ **(8)**

b. For any $A \in F(X)$, prove that the following property holds $\alpha_A = \bigcap_{\beta<\alpha}^{\beta} A = \bigcap_{\beta<\alpha}^{\beta+} A$ **(8)**

4. a. Let $f : X \to Y$ be an orbitrary crisp function. Then for any $B \in F(Y)$, prove that $f^{-1}(1-B) = 1 - f^{-1}(B)$ **(8)**

b. Let a function $C : [0,1] \to [0,1]$ satisfy axioms C_2 and C_4 of fuzzy complement. Then prove that c also satisfys the axioms C_1 and C_3 and also prove that C is a bijection function. **(8)**

5. a. Let i_w denote the class of Yager t-norms defined by

$i_w(a,b) = 1 - \min(1, [(1-a)^w + (1-b)^w)^{1/w}, w > 0$, then prove that

$i_{\min}(a,b) \leq i_w(a,b) \leq \min(a,b), \forall a,b \in [0,1]$ **(8)**

b. For all $a,b \in [0,1]$, prove that $i_{\min}(a,b) \leq i(a,b) \leq \min(a,b)$ where $i_{\min}$ denotes the drastic intersection. **(8)**

6. a. Let u_w denote the class of yager t-conorms defined by

$u_w(a,b) = \min(1, (a^w + b^w)^{1/w}, w > 0$, then prove that

$\max(a,b) \leq u_w(a,b) \leq u_{\max}(a,b), \forall a,b \in [0,1]$ **(8)**

b. Given a t-norm i and an involutive fuzzy complement c, then prove that the binary operation u on [0,1] defined by $u(a,b) = c(i(c(a), c(b))) \forall a,b \in [0,1]$ is a t-conorm such that $\langle i,u,c \rangle$ is a dual triple. **(8)**

7. a. Write a detailed note on linguistic variables. **(8)**

b. Define fuzzy arithmetic operations and explain with suitable examples. **(8)**

8. a. Define fuzzy relations and explain with suitable example **(4)**

b. What are projections and cylindrical extensions? Explain with example. **(6)**

c. Write a note on binary fuzzy relations. **(6)**

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PGIS-N 1020 B-18

M.A/M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

(Algebra - I)

Paper - HCT :1.2

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

Section - A

1. a) Let N be a normal subgroup of a group G and H a subgroup of G. Then prove that NH is a subgroup of G. **(8)**

b) Let G be a finite group and $a \in G$. Let N(a) denote the normalizer of a in G and Ga the conjugate class of a in G. Then show that $O(Ga) = \sum \frac{O(G)}{O\,N(a)}$ **(8)**

2. a) If p is a prime number and G is a non-abelian group of order P^3. Show that the centre of G has exactly p elements. **(8)**

b) Let H be a normal subgroup of a finite group G. Then prove that $O(G/H) = \frac{O(G)}{O(H)}$ **(8)**

3. a) Define external and internal direct product, p - subgroup and sylow p-subgroup. **(8)**

b) If $G = H_1 \otimes H_2$ if, G is the internal direct product of its subgroups H_1 and H_2, then prove that **(8)**

i) H_1 and H_2 are normal subgroups of G

ii) $G/H_1 = H_2, G/H_2 = H_1$

4. State and prove first Sylow Theorem. **(16)**

5. a) Prove that every integral domain can be embeded in a field. (8)

b) If F is a field, then prove that F[x] is a Euclidean domain (8)

6. a) Let R be a UFD. Then prove that the product of two primitive polynomials over R is also a primitive polynomial. (8)

b) Prove that the sub module S generated by N and K is the submodule $N+K=\{x+y/x\in N, y\in K\}$ (8)

7. a) State and prove Eiensteins Criteria (8)

b) Prove

$$F(\alpha_1,\alpha_2,..\alpha_n)=\left\{\frac{f(\alpha_1,\alpha_2,..\alpha_n)}{g(\alpha_1,\alpha_2,......\alpha_n)}/\right.$$

$$f,g\in F[x_1,x_2,.....x_n],$$ (8)

$$g(\alpha_1,\alpha_2,......\alpha_n)\neq 0\}$$

8. a) Let $f(x)\in F[x]$ be of degree n. Then prove that $f(x)$ has a splitting field. (8)

b) Prove that two elements α and α' are conjugate over F iff they have the same minimum polynomial over F. (8)

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PGIS-O 1021 B-18

M.A/M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

Algebra - I

Paper - HCT 1.2

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any Five questions*
2. *All questions carry equal marks.*

1. a) For a subset A of a group G, prove that A is normal subgroup of G iff N(A) = G. **(8)**

   b) Show that if G is a finite group, then $Ca = \frac{O(G)}{ON(a)}$ **(8)**

2. a) Let G be a finite group of order n. Then show that G is isomorphic to a subgroup of S_n. **(8)**

   b) Prove that G is a direct product of subgroups H and K iff

      i) Every $x \in G$ can be uniquely expressed as $x = hK,\ h \in H, k \in K$

      ii) $hk = kh, h \in H, k \in K$ **(8)**

3. a) Show that any subgroup H of a solvable group G is solvable. **(8)**

   b) Define Sylow's P - Subgroup of a group G with an example. **(8)**

4. Show that every integral domain can be embedded in a field. **(16)**

5. a) State and prove unique factorisation theorem. (8)

b) If R is a commutative ring with unit element, then show R[x] is also a commutative ring. If R is an integral domain show that R[x] is also an integral domain. (8)

6. a) Let R be a unique factorisation domain and F the quotient field of R. Let $f(x) \in R(x)$ be irreducible in R[x]. Then show that f(x) is also irreducible in F[x]. (8)

b) Let A be an ideal of a ring R and a be an element of A. Prove that A has exactly of all ay as y ranges over R. (8)

7. a) With usual notation prove that $[L:F]=[L:K][K:F]$ where L, K are extension fields of F. (8)

b) Show that a polynomial of degree n over a field has at most n roots in any extension F. (8)

8. a) Let $x \in F[x]$ be of degree n. Then show that K/F is an algebraic extension. (8)

b) Define perfect field. Let F be a field of characteristic $p(\neq 0)$. Show that an element a, in some extension of F, is separable over F iff $F(a^p)=F(a)$ (8)

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PGIS-N 1022 B-18

M.Sc. I Semester (CBCS) Degree Examination

MATHEMATICS

(Ordinary Differential Equations)

Paper - HCT 1.3

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Solve any **Five** questions
2. All questions carry **equal** marks.

1. a) For any real x_0 and constants α, β prove that there exists a solution ϕ of the initial value problem

$$L(y) = y'' + a_1 y' + a_2 y = 0, with\ y(x_0) = \alpha, y'(x_0) = \beta\ in\ -\infty < x < \infty \quad \textbf{(8)}$$

b) Let ϕ (x) be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ in an interval I containing a point x_0. Then for all x in I, Prove that.

$$\|\phi(x_0)\| e^{-K|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{K|x-x_0|} where$$

$$\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{1/2} \ \& \ K = 1 + |a_1| + |a_2| \quad \textbf{(8)}$$

2. a) Define linear dependence & independence further prove that two solutions. ϕ_1, ϕ_2 *of* $L(y)=0$ *are linearly* independent on an interval I iff $W(\phi_1,\phi_2)(x) \neq 0, \forall x \in I$ **(8)**

b) Let ϕ_1, ϕ_2 be any two linearly independent solutions of L(y) = 0 on an interval I. Then prove that every solution ϕ of L(y) = 0 can be written uniquely as $\phi = C_1\phi_1 + C_2\phi_2$, *where* C_1, C_2 are constants **(8)**

3. a) Define adjoint and self adjoint equation. Futher show that the following equations are self-adjoint and write the equation. **(8)**

i) $\sin x.y'' + \cos x.y' + 2y = 0$

ii) $x^3.y'' + 3x^2.y' + y = 0$

b) Let $u(x) \& v(x)$ be two solutions of $[r(x)y']' + p(x)y = 0$, such that $u(x) \& v(x)$ have a common zero on $a \le x \le b$. Then, prove that $u(x) \& v(x)$ are linearly dependent on $a \le x \le b$. **(8)**

4. a) State and prove sturm seperation theorem **(8)**

b) Show that the zeros of the functions a $\sin x + b\cos x$ *and* $c\sin x + d\cos x$ are distinct and occur alternately whenever $ad - bc \neq 0$ **(8)**

5. a) Use the power series method to find the general solution of $(1-x^2)y'' + 2y = 0$, *with* $y(0) = 4, y'(0) = 5$ **(8)**

b) Define ordinary and singular points. Further, explain the method of solving second order linear differential equation for which x = 0 is an ordinary point. **(8)**

6. a) Explain working rule of Frobenins method. **(8)**

b) Find the power series solution of the initial value problem $xy'' + y' + 2y = 0$, *in powers of* $(x-1)$ with initial condition $y(0) = 2, y'(0) = 4$ **(8)**

7. a) Define : **(8)**
i) Orthogonality
ii) Orthogonal set of functions
iii) Orthonormal set of functions

b) Explain Gram - Schmidt process of orthonormalization. **(8)**

8. a) Prove that eigen functions corresponding to different eigen-values are othogonal with respect to some weight function **(10)**

b) Find the eigen values and the corresponding eigen functions of $x'' + \lambda x = 0 \; with \; x(0) = 0, x'(L) = 0$ **(6)**

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PGIS-O-1023 B-18

M.Sc. I Semester Degree Examination

MATHEMATICS

(Ordinary Differential Equations)

Paper - HCT 1.3

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Solve any* ***FIVE*** *questions.*
2. *All questions carry* ***equal*** *marks.*

(5×16=80)

1. a. If $\phi_1, \phi_2 ...., \phi_n$ are n solutions of L (y) = 0 on I, then they are linearly independent iff $W(\phi_1, \phi_2 ...., \phi_n)(x) \neq 0 \; \forall \; x$. **(8)**

   b. If $\phi_1, \phi_2 ...., \phi_n$ are n solutions of L (y) = 0 on an interval I and x_0 be any real point on I, then prove that

$$W(\phi_1, \phi_2, ..., \phi_n)(x) = \exp.\left[-\int_{x_0}^{x} a_1(t)dt\right] \times W(\phi_1, \phi_2, ..., \phi_n)(x_0)$$ **(8)**

2. a. If $\alpha_1, \alpha_2, ..., \alpha_n$ are any n constants and x_0 be any real number, then $\exists$ a solution ϕ of L (y) =0 $-\infty < x < \infty$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, ..., \phi^{(n-1)}(x_0) = \alpha_n$ **(8)**

   b. If ϕ_1, ϕ_2 are two solutions of L (y) =0 in an interval I containing point x_0 then prove that $W(\phi_1, \phi_2)(x) = e^{-a'(x-x_1)} . W(\phi_1, \phi_2)(x_0)$ **(8)**

3. a. Define Green's function. Further, explain the method of finding Green's function. **(8)**

   b. Find the Green's function corresponding to differential operator $L = \frac{d^2}{dx^2}$ with boundary Conditions y(0) =0, $y'(1)$=0. **(8)**

(8)

4. a. State and prove sturm comparision theorem. **(8)**

b. Show that the boundary value problem $\frac{d^2x}{dt^2}+\lambda x=0$ where x (0) = 0, $x(\pi)=0$, is sturm Liouville problem. **(8)**

5. a. Define adjoint and self-adjoint equation. Further, transform the following equation into an equivalent self- adjoint equation

$$x^2y^{11}-2xy^1+2y=0$$ **(8)**

b. Find the approximate solution of $y^1=x(y-x^2+z)$, with y(0) =1 by the method of successive approximation. **(8)**

6. a. State and prove existence theorem for system of differential equations. **(8)**

b. Using the method of Picard, obtain third approximation of the solution of the equation $y^1=2y-2x^3-3$, where y = 2 when x = 0. **(8)**

7. a. Derive the method of obtaining general solution of Riccatti's equation. **(8)**

b. Prove that the cross-ratio of any four particular integrals of a Riccatti's equation. **(8)**

8. a. Show that there are two values of the constant k for which k/x is an integral of $x^2(y_1+y^2)=2$, & hence obtain the general solution. **(10)**

b. Solve the equation $x^2y_1+2-2xy+x^2y^2=0$ by Riccatti's method. **(6)**

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PGIS-O 1025 B-18

M.Sc. I Semester Degree Examination

MATHEMATICS

(Discrete Mathematics)

Paper - HCT 1.4

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **FIVE** questions.
2. All questions carry **equal** marks.

1. a. Define the following and write an example of each
   i. Binary relation
   ii. Poset
   iii. Least upper bound
   iv) Greatest Lower bound (4)

   b. If L is lattice, then for any a,b and c in L, prove that
   i. $a \vee (b \vee c) = (a \vee b) \vee c$
   ii. $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ (6)

   c. Let L be a bounded distributive lattice. If a complement exists then prove that it is unique. (6)

2. a. If $(L, \leq)$ is a Boolean algebra, then for all a and b in L, prove that
   i. $(a \vee b)^1 = a^1 \wedge b^1$
   ii. $(a \wedge b)^1 = a^1 \vee b^1$ (6)

   b. Let $(A, \vee, \wedge, -)$ be a finite Boolean algebra. Let b be any non zero element in A, and a_1, a_2, ----- a_k be all the atoms of A such that $a_i \leq b$. Then prove that

   $b = a_1 \vee a_2 \vee ..... \vee a_k$ (6)

c. Design a switching circuit corresponding to the compound statement.

$(a \wedge b) \vee (a \wedge c) \vee (b \wedge c)$ **(4)**

3. a. Determine how many distinguishable permutations of the letters in the word BANANA. **(4)**

b. For any two finite sets S and T, prove that

$$|S U T| = |S| + |T| - |S \cap T|$$ **(6)**

c. A shopkeeper sells only onion, carrots and potato's one day, the shopkeeper served 208 people. If 114 purchased onions, 152 purchased carrots, 17 purchased potatoes, 64 purchased onion and carrot, 12 purchased carrots and potato's and 9 purchased all three. Determine how many purchased onion and potato. **(6)**

4. a. Solve the recurrence relation of Fibonacci sequence of numbers $a_r = a_{r-1} + a_{r-2}$ with $a_0 = 1$ and $a_1 = 1$. **(8)**

b. Solve the recurrence relation

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$ **(8)**

5. a. Define the following
   i. Undirected graph
   ii. Directed graph
   iii. Multi graph
   iv. Eulerian graph
   v. degree of a vertex **(5)**

b. Prove that, the number of vertices of odd degree in a graph is always even **(6)**

c. Write a note on adjacency matrix. **(5)**

6. a. Prove that, there is one and only one path between every pair of vertices in a tree T. **(6)**

b. Prove that a tree with n-vertices has (n-1) edges **(6)**

c. Write a note on transport network. **(4)**

7. a. Define the following
   i. Semi group
   ii. Monoid
   iii. Group
   iv. Sub group

   Give an example of each. **(4)**

b. If every element of group (G, *) is its OWN inverse, then prove that G is abelian. **(6)**

c. State and prove Lagrange's theorem. **(6)**

8. a. Write note on coding of binary information and error detection. **(4)**

b. If x, y and z be elements of B^m, then show that

i. $\delta(x,y) = \delta(y,x)$

ii. $\delta(x,y) \geq 0$

iii. $\delta(x,y) \leq \delta(x,z) + \delta(z,y)$ **(6)**

c. An (m, n) encoding function e; $B^m \to B^n$ can detect k or fewer errors if and only if its minimum distance is at leas (k+1). **(6)**

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PGIS-N 1024 B-18
M.Sc. I Semester Degree Examination
MATHEMATICS
(Discrete Mathematics)
Paper - HCT 1.4
(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. *Answer any **FIVE** questions.*
2. *All questions carry **equal** marks.*

1. a. What are rules of inference? Discuss various rules of inferences for constructing valid arguments from the given statements. **(4)**

   b. Let n is an integer. Prove that if n^2 is odd, then n is odd. **(6)**

   c. Show by mathematical induction that for all $n \geq 1$, $1+2+..........+n = \frac{n(n+1)}{2}$. **(6)**

2. a. How many distinguishable permutations of the letters in the word BANANA are there? **(4)**

   b. For any two finite sets S and T, prove that $|S \cup T| = |S| + |T| - |S \cap T|$ **(6)**

   c. Thirty cars were assembled in a factory. The options available were a radio, an air conditioner and white wall tires. It is known that, 15 of the cars have radios, 8 of them have air conditioners and 6 of them have white-wall tires.

   More over, 3 of them have all 3 options. We want to know at least how many cars do not have any options at all. **(6)**

3. a. Obtain the generating function corresponding to the numeric function $a_r = 3^{r+2}, r \geq 0$. **(4)**

   b. Obtain the numeric function corresponding to the generating function $A(z) = \frac{z^4}{(1-2z)}$. **(4)**

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c. Solve the recurrence relation $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$. (8)

4. a. Solve the recurrence relation $a_r + a_{r-1} = 3r2^r$. (6)

b. Solve the recurrence relation $a_r = 3a_{r-1} + 2, r \geq 1$ with the boundary condition $a_0 = 1$, by the method of generating function. (10)

5. a. Define the following and explain with suitable example.

i. Reflexive binary relation

ii. Partially orderd set

iii. Maximal element

iv. Minimal element (4)

b. If L be a lattice, then prove that

i. $a \vee a = a$

ii. $a \wedge a = a$ (6)

c. Let L be a bounded distributive lattice. If a complement exists, then prove that it is unique. (6)

6. a. If the meet operation is distributive over the join operation in a lattice, then prove that, the join operation is also distributive over the meet operation. (8)

b. If $(L, \leq)$ is a Boolean algebra, then for all a and b in L prove that

i. $(a')' = a$

ii. $(a \wedge b)' = a' \vee b'$ (8)

7. a. Define the following and write an example of each.

i. Semigroup

ii. Monoid

iii. abelian group

iv. Group (4)

b. For any commutative monoid (M, *), prove that,the set of idompotent elements of M forms a submonoid. (6)

c. State and prove Lagranges theorem. (6)

8. a. Write a detailed note on coding of binary information and error detection.

b. Find Hamming distance between code works x and y in B^6

i. x=110110; y=000101

ii. x=001100; y=010110

c. Consider the (3,8) encoding function

$e : B^3 \to B^8$ defined by

$e(000) = 00000000$
$e(001) = 10111000$
$e(010) = 00101101$
$e(011) = 10010101$
$e(100) = 10100100$
$e(101) = 10001001$
$e(110) = 00011100$
$e(111) = 00110001$

How many errors will e detect? **(16)**

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PGIS-O 1029 B-18
M.Sc. I Semester Degree Examination
MATHEMATICS
(General Topology)
Paper - HCT 1.5
(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

*i. Answer any **five** full questions.*

*ii. All questions carry **equal** marks.*

(5×16=80)

1. a. Define closure of a set. Let A and B be subsets of a space X then prove the followings:

   i. $\overline{A}$ is the smallest closed set containing A.

   ii. A is closed iff A=$\overline{A}$

   iii. If $A \subset B$ then $\overline{A} \subset B$

   iv. $\overline{A \cup B} = \overline{A} \cup \overline{B}$

   v. $\overline{(\overline{A})} = \overline{A}$ **(8)**

   b. Let X be any topological space. Using Kuratowski's closure axioms, Prove that there exists a topology τ on X such that $C(A) = \overline{A}$. **(8)**

2. a. Define a limit point of a set. If A & B are subsets of a space X then Prove the followings:

   i. If $A \subset B$ then $D(A) \subset D(B)$

   ii. $D(A \cup B) = D(A) \cup D(B)$ **(8)**

   b. Define a base for a topology. Then prove the following two properties on a base β are equivalent:

   i. β is a base for τ

   ii. For each $G \in \tau$ and each $p \in G$ there is $U \in \beta$ such that $p \in U \subset G$. **(8)**

[Contd....

(11)

3. a. Define a continuous mapping. If $f: X \to Y$ be a continuous function & $A \subset X$ then prove that $f/A: A \to Y$ is continuous. (8)

b. Define a closed mapping. If X & Y are the topological spaces and $f: X \to Y$ be a mapping then prove that $\overline{f(A)} \subset f(\bar{A})$ for $A \subset X$. (8)

4. a. Prove that a space (X, τ) is a To-space iff for each pair of distinct points $x, y \in X, \overline{\{x\}} \neq \overline{\{y\}}$. (8)

b. Define a T_2- space. If a topological space X is T_2 and $f: X \to Y$ is a closed bijection then show that Y is also a T_2- space. (8)

5. a. Prove that $X \times Y$ is a regular iff X & Y are both regular spaces (8)

b. Show that every 2^0- countable space is 1^0- countable. Is the converse true? Justify your answer with an example (8)

6. a. Let X be a 1^0- countable space and $A \subset X$. Then prove that $a \in X$ is a limit point of A iff there exists a sequence $f: N \Rightarrow A - \{a\}$ converging to 'a'. (8)

b. Prove that any continuous image of a connected space is connected. (8)

7. a. Define a compact space. Let A be a compact subset of Hausdorff space X and $p \notin A$. Then prove that there exists disjoined open sets U and V such that $p \in V$ and $A \subset U$ (8)

b. Prove that the space X is compact iff every collection of closed sets with FIP has a non empty intersection. (8)

8. a. Define a Metric space. Prove that every separable metric space (X,d) is 2^0- countable. (8)

b. Define a Lindelof space. Show that every closed subspace of Lindelof space is Lindelof. (8)

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