Roll No. ______________________________ [Total No. of Pages : 2

PGIS-O 1029 B-17
M.A/M.Sc. Ist Semester (CBCS) Degree Examination
Mathematics
(General Topology)
Paper : HCT - 1.5
(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) Define a topology J on a non - empty set X. Let μ consist of ϕ and all those subset G of a real line $\mathbb{R}$ such that to each $x \in G$ there exists $\in > 0$ such that $(x-\in, x+\in) \subset G$. Then show that μ is a topology on $\mathbb{R}$. **(8)**

   b) Using the Kuratowski's closure axioms in a topological space X, then prove that there exists a topology J on X such that $C(A) = \overline{A}$. **(8)**

2. a) Define limit point a set. If A & B are subsets of a space X then prove the followings:

   i) If $A \subset B$ then $D(A) \subset D(B)$

   ii) $D(A \cup B) = D(A) \cup D(B)$ **(8)**

   b) Let (X,J) be a topological space and β is a subfamily of J then prove the following two properties B are equivalent :

   i) β is a base for J

   ii) for each $G \in J$ and each $P \in G$ there is $U \in \beta$ such that $P \in U \subset G$. **(8)**

3. a) If $f : X \to Y$ be a continuous function and $A \subset X$ then show that $f / A : A \to Y$ is continuous. Is the converse true? Justify. **(8)**

b) If $f: X \to Y$ be a map where X and Y are topological spaces. Then prove the following properties are equivalent :

i) f is an open mapping

ii) $f(A^\circ) \subset (f(A))^\circ$ for each $A \subset X$.

iii) The image of each member of basis for X is an open set in Y.

iv) For each $x \in X$ and each neighborhood W of f(x) in Y so that $f(x) \in W \subset f(U)$. **(8)**

4. a) Define a Regular space. Prove that Regularity is a topological property. **(8)**

b) Define a connected space. Show that continuous image of a connected space is connected. **(8)**

5. a) State the two axioms of countability with an example. Prove that every second countable space is first countable. **(8)**

b) If X be 1° - countable space and $A \subset X$ then show that for $a \in X$ is a limit point of A iff there exists a sequence $f: N \to A - \{a\}$ converging to 'a'. **(8)**

6. a) Define a compact space. Show that every compact subset of a Hausdorff space X is closed. **(8)**

b) Show that a topological space is compact if and only if every collection of closed sets with finite intersection property has a non - empty intersection. **(8)**

7. a) If X be a compact Hausdorff space and Y be an arbitrary space. If $f: X \to Y$ is a continuous closed surjection then prove that Y is also a Hausdorff space. **(8)**

b) Let X and Y be spaces. Then prove that X×Y is separable if and only if X and Y are both separable. **(8)**

8. a) Prove that every regular T_1 - space X satisfying the second axiom of countability is metrizable. **(8)**

b) Define a Lindelof space. Show that every closed subspace of Lindelof space is Lindelof. **(8)**

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PGIS-N-1029 B-17

M.A. / M.SC. Ist Semester (CBCS) Degree Examination

MATHEMATICS

(General Topology)

Paper : HCT - 1.5

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates :

i. *Answer any* ***Five*** *full questions.*

ii. *All questions carry equal marks.*

1. a) Define a topology on X. Give two examples of different types of topology. **(6)**

b) Define closure of a set. Let A and B be subsets of a space x then prove the following:

i) $\overline{A}$ is the smallest closed set containing A.

ii) A is closed iff $A=\overline{A}$

iii) If $A\subset B$ then $\overline{A}\subset\overline{B}$

iv) $\overline{A\cup B}=\overline{A}\cup\overline{B}$

v) $\overline{\left(\overline{A}\right)}=\overline{A}$ **(10)**

2. a) Define limit point of a set . If A is a subset of a space x then prove the followings:**(8)**

i) $A\cup D(A)$ is closed

ii) $\overline{A}= A\cup D\ (A)$ **(8)**

b) Define a continuous function. If $f{:}x\rightarrow y$ is a continuous function and $A\subset X$ then prove that f/A : $A\rightarrow y$ is also continuous. **(8)**

3. a) Define a T_0-space. show that a space (X,J) is a T_0- space iff for each pair of distinct points x, y ε x, $\overline{\{x\}}\neq\overline{\{y\}}$. **(8)**

b) Prove that XxY IS T_2- space iff X & Y are both T_2- spaces. **(8)**

4. a) Define a Regular space. Prove that regularity is hereditory. **(8)**

b) Prove the following 4 properties of T_4- space are equivalent:

i) X is Normal.

ii) for each closed set A and an open set U with $A \subset U$ there exists an open set V such that $A \subset V \subset \overline{V} \subset U$

iii) for each pair disjoined closed sets A, B in X there exists an open set U such that $A \subset U$ & $\bar{U} \cap B = \phi$

iv) for each pair of disjoined closed sets A,B in x there exists open sets U, V such that $A \subset U,\ B \subset V$ and $\bar{U} \cap \bar{V} = \phi$ **(8)**

5. a) State the two axioms of countability show that a continuous image of a separable space is separable. **(8)**

b) Define subsequence of a sequence let f : N →x converges to 'a' in x then prove that every subsequence of 'f' in x converges to 'a'. **(8)**

6. a) Define a connected space. let $\{A_\alpha : \alpha \in D\}$ be a family of connected subsets of a space x such that one of the members of this family intersects every other member then prove that $U\{A_\alpha : \alpha \in D\}$ is connected. **(8)**

b) Define component of a connected space prove the followings.

i) Every component of a space x is a maximal connected set.

ii) The components are closed. **(8)**

7. a) State and prove Heine-Borel theorem. **(10)**

b) Prove that the space x is compact iff every collection of closed sets with FIP has a non empty intersection. **(6)**

8. a) Define a metric space. Show that every metric space is a Housdorff space. **(8)**

b) Define a Lindelof space. Show that a metric space is Lindelof iff it is second countable. **(8)**

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PGIS-1026 B-17

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

MATHEMATICS

(Classical Mechanics)

Paper : SCT 1.1

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to the Candidates :

*i) Answer any **five** questions.*

*ii) All questions carry **equal** marks.*

1. a) State and prove D'Alemberts principle. **(8)**

   b) Derive Lagrange's equation for impulsive motion. **(8)**

2. a) Construct Lagrangian and hence equation of motion of a simple pendulum placed in a uniform gravitational field. **(8)**

   b) Derive energy equation for impulsive motion. **(8)**

3. a) Derive the HAMILTON's canonical equation from Lagrangian variables. **(8)**

   b) Define rigid body. Find the momentum inertia and product inertia of a body at 0. **(8)**

4. a) Explain transformations associated with Eulerian angles. **(8)**

   b) State Hamilton's principle of least action and derive Hamilton's canonical equation from Hamilton's principle. **(8)**

5. a) Obtain Hamilton-Jacobi equations for simple harmonic motion and find a complete integral and determine solution of it. **(8)**

   b) State lee-Haw-Chung theorem and derive Whittaker's equations. **(8)**

6. a) Define cyclic Coordinates. Write steps of Routh's procedure to a problem with cyclic and remaing non-cyclic Coordinates. **(8)**

   b) Prove that if F, G are both integrals of motion, then so is their Poisson-bracket. **(8)**

7. a) Show that Poisson's bracket's are also invariant under canonical transformation. (8)

b) Given the generating function

$F_1(q,Q,t)=\frac{1}{2}mw\left[q-\frac{F(t)}{mw^2}\right]^2\cot Q$. Find transformation equations and thus obtain equations of motion of a simple harmonic oscillator acted upon by a force F(t) in terms of Q and P. (8)

8. a) Show that Lagrange's bracket donot obey the commutative law. Also prove the fundamental Lagranges bracket. (8)

b) Find the relationship between. Lanrange's and Poisson's bracket. (8)

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PGIS-1027 B-17

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

MATHEMATICS

(Operation Research)

Paper : SCT 1.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to the Candidates :

*i) Answer any **five** questions.*

*ii) All questions carry **equal** marks.*

1. a) Explain briefly the general methods for solving OR models. **(8)**

   b) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of cloths namely, suitings, shirtings and woolens yielding the profit of Rs. 2, Rs. 4 and Rs. 3 per meter respectively. One meter suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. One meter of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing, while one meter woolen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours of weaving, processing and packing departments respectively. Formulate the problem as a linear programming problem. **(8)**

2. a) Use graphical method to solve the following L.P.P **(8)**

   Minimize $Z = 20x_1 + 40x_2$

   Subject to

   $$36x_1 + 6x_2 \geq 108$$

   $$3x_1 + 12x_2 \geq 36$$

   $$20x_1 + 10x_2 \geq 100$$

   and $x_1, x_2 \geq 0$

b) Solve the following L.P.P. by simplex method (8)

Maximize $Z = -x_1 + 3x_2 - 2x_3$

Subject to

$$3x_1 - x_2 + 3x_3 \le 7$$

$$-2x_1 + 4x_2 \le 12$$

$$-4x_1 + 3x_2 + 8x_3 \le 10$$

and $x_1, x_2, x_3 \ge 0$

3. a) Use Big-M method to solve the following LPP. (8)

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

and $x_1, x_2, x_3, x_4 \ge 0$

b) Use two-phase simplex method to solve the following LPP. (8)

Maximize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$x_1 - x_2 + x_3 \ge 4$$

$$x_1 + x_2 + 2x_3 \le 8$$

$$x_1 - x_3 \ge 2$$

and $x_1, x_2, x_3 \ge 0$

4. a) Prove that dual of the dual is primal. (8)

b) Use dual simplex method to solve the following LLP. (8)

Minimize $Z = 3x_1 + x_2$

Subject to

$$x_1 + x_2 \ge 1$$

$$2x_1 + 3x_2 + \ge 2$$

and $x_1, x_2 \ge 0$

5. a) Explain the method of row minima and column minima to obtain initial basic feasible solution to the given transportation problem. (10)

b) Determine the initial basic feasible solution to the following transportation problem by using vogel's approximation method. (6)

(Destinations)

		A	B	C	D		
	I	21	16	25	13	11	
(Origins)	II	17	18	14	23	13	(Availability)
	III	32	27	18	41	19	
		6	10	12	15		

(Requirements)

6. a) Write the transportation technique. (4)

b) Find the optimum transportation schedule for the following problem by using transportation technique. (12)

(Destinations)

		D_1	D_2	D_3	D_4		
	O_1	10	0	20	11	15	
(Source)	O_2	12	7	9	20	25	(Supply)
	O_3	0	14	16	18	5	
		5	15	15	10		

(Demand)

7. a) Write the algorithm of the assignment problem (8)

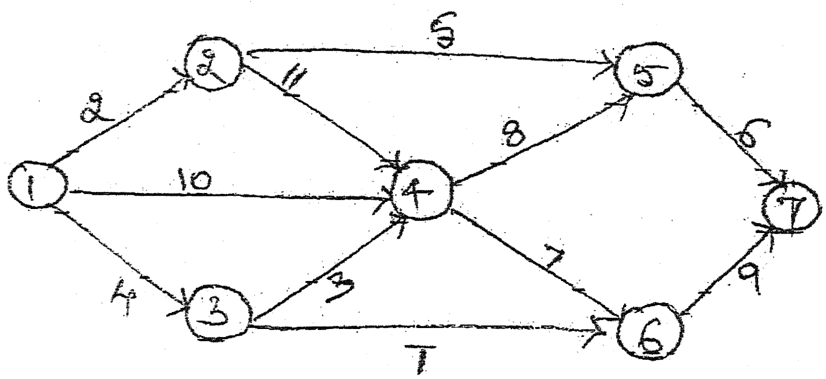
b) Solve the following assignment problem. (8)

(Machines)

		I	II	III	IV
	A	2	3	4	5
(Jobs)	B	4	5	6	7
	C	7	8	9	8
	D	3	5	8	4

8. a) In a game of matching coins with two players, the matching player is paid Rs. 8.00 if the two coins turn both heads and Rs. 1.00 if the coin turn both tails. The non-matching player is paid Rs. 3.00 when the two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy. **(8)**

b) For the following network, find the shortest route from node 1 to node 7. **(8)**



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PGIS-O 1025 B-17

M.A/M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Discrete Mathematics)

Paper : HCT - 1.4

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) If L is a lattice, then for any a, b and c in L, prove the following

i) $a \vee (b \vee c) = (a \vee b) \vee c$

ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

iii) $a \vee (a \wedge b) = a$ **(6)**

b) Prove that in a distributive lattice if an element has a complement then it is unique. **(5)**

c) If join operation is distributive over the meet operation then prove that the meet operation is also distributive over the join operation. **(5)**

2. a) Design the digital network for the Boolean expression $y = \left[(x_1 \vee x_2) \wedge (x_1^1 \wedge x_3)\right] \vee (x_2 \vee x_3)$ and also write logic table for the corresponding combinatorial circuit. **(6)**

b) Using Boolean algebra techniques simplify the Boolean expression $f(x, y) = x \vee (x \wedge y) \vee y$ **(5)**

c) Draw the switching network corresponding to the algebraic expression *xSER ((y SERz) PAR (z SER(ePARf)))* **(5)**

3. a) Suppose that a valid computer password consists of seven characters, the first of which is a letter chosen from the set {A,B,C,D, E,F,G} and the remaining Six characters from the English alphabet or digit. How many different passwords are possible **(5)**

b) For any two finite sets A and B, prove that $|A \cup B| = |A| + |B| - |A \cap B|$ (6)

c) Using the principle of inclusion and exclusion determine the number of integers between 1 and 250 that are divisible by any of the integers 2,3,5 and 7. (5)

4. a) Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = 0$ with $a_0 = 1$ and $a_1 = 6$. (8)

b) Solve the recurrence relation $a_r = 3a_{r-1} + 2, r \geq 2$ with the boundary conditions $a_0=1$ by the method of generating function. (8)

5. a) Define the following and give an example of each.

i) Weighted graph

ii) Multi graph. (4)

b) Prove that a given connected graph 'G' is Euler graph if and only if all vertices of G are of even degree. (6)

c) Prove that in a simple graph with n - vertices and K components can have atmost (n-K)(n-K+1)/2 edges (6)

6. a) Write a note on tree (5)

b) If a binary tree of height h has 't' terminal vertices, then prove that $\log_e t \leq h$ (6)

c) Write a note on state transition graphs. (5)

7. a) Define semigroup and monoid. Further, if f is an isomorphism from a semigroup (S,*) to semigroup (T, *) then prove that f^{-1} is an isomorphism from (T,*) to (S,*).(6)

b) Prove that a non empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$ (5)

c) State and prove Lagranges theorem on groups. (5)

8. a) Find the length and weights of the following code words.

i) $x = 00111$

ii) $x = 01010$

iii) $x = 101001$

iv) $x = 00111$ (4)

b) Prove that an encoding function $e: B^m \to B^n$ can detect k or fewer errors if and only if its minimum distance is at least (k+1). (6)

c) Write a note on coding of binary information. (6)

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PGIS-N-1025-B-17

M.A/M.Sc Ist Semester (CBCS) Degree Examination

MATHEMATICS

(Discrete Mathematics)

Paper : HCT 1.4

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

i. Answer any five questions

ii. All questions carry equal marks

1. a) What are logic variables. Discuss various logic connectives used in forming a compound statement **(8)**

b) What is a predicate? Discuss the two types of quantifications used for extending the scope of a predicate **(8)**

2. a) Determine the number of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and the remaining once white **(5)**

b) State and prove the principle of inclusion-exclusion for two sets **(6)**

c) A software firm wants to hire 25 programmers to work on systems programming project and 40 programmers for application programming. Of those hired ten will be expected perform both types of jobs. How many programmers must be hired. **(5)**

3. a) write a note on discrete numeric functions **(5)**

b) Obtain the generating function of $a_r = 3^{r+2},\ r \geq 0$ **(5)**

c) Solve the recurrence relation $a_r = a_{r-1} + a_{r-2}$ with $a_o = 1$ *and* $a_{1=1}$ **(6)**

4. a) Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ *with* $a_o = 0$ *and* $a_1 = 1$ **(4)**

b) Find the particular solution of $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$ **(4)**

c) Solve the recurrence relation $a_r = 3a_{r-1} + 2$, $r \geq 1$ with the boundary conditions $a_0 = 1$ by the method of generating functions **(8)**

5. a) Define a binary relation and discuss the properties of binary relations with suitable example (5)

b) Define a lattice and for any a,b,c,d in a lattice $(\angle,\leq)$ if a $\leq$b and c$\leq$d then prove that

i) $a \vee c \leq b \vee d$

ii) $a \wedge c \leq b \vee d$ (6)

c) Let L be a lattice then for every a and b in L prove that $(a \wedge b) = a \; iff \; a \vee b = b$ (5)

6. a) If $(L,\leq)$ be a distributive lattice then for any a,b,c $\in$ L if $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ prove that b=c (5)

b) Draw the combinatorial circuit for the boolean expression

$y_1 = [(x_1 \wedge x_2) \vee x_3] \wedge (x_2 \vee x_3)$ and write the logic table for the corresponding combinatorial circuit (5)

c) Let $(A, \vee, \wedge, -)$ be a finite boolean algebra. let b be any non zero element in A, and $a_{1,} a_2 \; a_k$ be all the atoms of A such that $a_i \leq b$. Then prove that

$b = a_1 \vee a_2 \vee .... \vee a_k$ (6)

7. a) Define the group. If G is a group then prove that

i) a*b=a*c $\Rightarrow b = c$

ii) b*a=c*a $\Rightarrow b = c$ (6)

b) Define a subgroup. Prove that a non-empty subset if of a group G is a subgroup of (G,*) iff a,b $\in$ H $\Rightarrow a * b^{-1} \in H$ (5)

c) If (G,*) is a finite group and (H,*) is a subgroup then prove that the order of H is a divisor of order of G (5)

8. a) Write a detailed note on group codes (5)

b) Find the weight of each of the following code words in B^5 (4)

i) x=11001

ii) x=11111

iii) x=01011

iv) x=10101

c) Prove that an encoding function $e : B^m \rightarrow B^n$ can detect K or fewer errors if and only if its minimum distance is at least (k+1) (7)

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PGIS-O 1023 B-17

M.A/M.Sc. Ist Semester (CBCS) Degree Examination

MATHEMATICS

(Ordinary Differential Equations)

Paper : HCT - 1.3

(Old)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) For any real x_0 and constants α, β show that there exists a solution of the initial value problem

$$y'' + a_1 y' + a_2 y = 0$$

$$y(x_0) = \alpha, y'(x_0) = \beta \text{ on}$$

$$-\infty < x < \infty$$ **(8)**

b) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if & only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I. **(8)**

2. a) if $\phi(x)$ be any solution of $y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0, then prove that for all x in I $\|\phi(x_0)\| e^{-k|x - x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{k|x - x_0|}$

where $k = 1 + |a_1| + |a_2|$ & $\phi(x) = \left\{|\phi(x)|^2 + |\phi'(x)|^2\right\}^{1/2}$ **(8)**

b) If ϕ_1, ϕ_2 are two solutions of $y'' + a_1 y' + a_2 y = 0$ in an interval I containing a point x_0 then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x - x_0)} W(\phi_1, \phi_2)(x_0)$. **(8)**

3. a) Define Green's function. Explain the method of finding Green's function (10)

b) Let $f(x)$ and $g(x)$ be any two solutions of $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right]+q(x)y=0$ on the interval $a \le x \le b$, then show that for all x on $a \le x \le b$

$p(x)\left[f(x)g'(x)-f'(x)g(x)\right]=k$ where k is constant. (6)

4. a) State and prove sturm's separation theorem. (8)

b) Show that the boundary value problem $\frac{d^2x}{dt^2}+\lambda x=0$ where $x(0)=0,\ x(\pi)=0$ is a sturm Liouville problem. (8)

5. Find the characteristic values and characteristic functions of the sturm Liouville problem.

a) $\frac{d^2y}{dx^2}+\lambda y=0$, with $y(0)-y'(0)=0$, $y(\pi)-y'(\pi)=0$ (8)

b) $\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0$ with $y'(1)=0, y'(e^{2\pi})=0, \lambda \ge 0$ (8)

6. a) Derive the method of successive approximations for the initial value problem $y'=f(x,y), y(x_0)=y_0$ (8)

b) Find the approximate solution of $y'=x(y-x^2+z)$, with $y(0)=1$ by the method of successive approximation. (8)

7. a) Using the method of picard, obtain the third approximation of the solution of the equation $y'=2y-2x^3-3$, where $y=2$, when $x=0$. (8)

b) Derive the method of obtaining general solution of Riccatti's equation. (8)

8. a) Derive the method for solving the Riccatti's equation when its one parameter integral is known. (6)

b) Show that there are two values of the constant K for which K/x is an integral of $x^2(y_1+y^2)=2$, and hence obtain the general solution. (10)

Roll No. ______________________ [Total No. of Pages : 3

PGIS-N-1022 B-17

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

MATHEMATICS

(Ordinary Differential Equations)

Paper : HCT 1.3

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to the Candidates :

i) *Answer any **Five** questions.*

ii) *All questions carry **Equal** marks.*

1. a) For any real x_0 and constants α,β show that there exists *a* solution of the initial value problem. $y''+a_1 y'+a_2 y=0$ with $y(x_0)=\alpha, y'(x_0)=\beta$ on $-\infty<x<\infty$. **(8)**

   b) If ϕ_1, ϕ_2 are two solutions of $y''+a_1 y'+a_2 y=0$ in an interval I containing a point x_0 then prove that $W(\phi_1,\phi_2)(x)-e^{-a_1}(x-x_0).\ W(\phi_1,\phi_2)(x_0)$. **(8)**

2. a) Let ϕ be any solution of $L(y)=y^{(n)}+\alpha_1 y^{(n-1)}+...+a_n y=0$ on an interval I containing a point x_0. Then prove that for all x in I.

$$\|\phi(x_0)\|e^{-k|x-x_0|}\le\|\phi(x)\|\le\|\phi(x_0)\|e^{k|x-x_0|} \text{ where}$$

$$\|\phi(x)\|=\left[|\phi(x)|^2+|\phi^1(x)|^2+...+|\phi^{(n-1)}|^2\right]^{1/2}$$ **(8)**

   b) Let $\phi_1, \phi_2, ..., \phi_n$ be a set of n linearly independent solutions of n^{th} - order homogeneous linear differential equation $a_o y^{(n)}+a_1 y^{(n-1)}+....+a_n y=0$ on $a\le x\le b$; Let W denote the Wronskian of $\phi_1, \phi_2, ..., \phi_n$ & let $x_0\in[a,b]$, then prove that

$$W(x)=W(x_0)\exp\left[-\int_{x_0}^{x}\frac{a_1(t)}{a_0(t)}dt\right] \text{ for all } x \text{ on } a\le x\le b.$$ **(8)**

3. a) Define the adjoint equation and self adjoint equation. (6)

b) Transform the following into an equivalent self adjoint equation : (10)

i) $x^2 y'' - 2xy' + 2y = 0$

ii) $f(x)y'' + g(x)y' = 0$

4. a) State and prove sturm separation theorem. (8)

b) State and prove sturm comparision theorem. (8)

5. a) Find the ordinary and singular point of the differential equation

$(x-1)y'' + xy' + \frac{1}{x}y = 0$ (5)

b) Show that $x = 0$ and $x = -1$ are singular points of the differential equation $x^2(x+1)^2 y'' + (x^2 - 1)y' + 2y = 0$ (5)

c) Define the following : (6)

i) Analytic function

ii) Ordinary point of $y'' + p(x)y' + Q(x)y = 0$

iii) Singular point of $y'' + P(x)y' + Q(x)y = 0$

6. a) Explain power series solution about an ordinary point. (8)

b) Find the power series solution of the differential equation $y'' + xy' + x^2 y = 0$ in powers of x (that is about $x_0 = 0$) (8)

7. a) Show that the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0$, where $x(0) = 0, x(\pi) = 0$ is a sturm Liouville problem. (8)

b) Define characteristic values and characteristic functions. Further, find characteristic values and characteristic functions of $\frac{d}{dx}\left[x \cdot \frac{dy}{dx}\right] + \frac{\lambda}{x}y = 0$ with $y'(1) = 0$, $y'\left(e^{2\pi}\right) = 0$, $\lambda \geq 0$. (8)

8. a) Define the following : (6)

i) Orthogonal set of functions

ii) Orthonormal set of functions

b) Obtain the formal expansion of the function f, where $f(x)=\pi x-x^2, 0\le x\le \pi$, in the series of orthonormal characteristic function $\{\phi_n\}$ of the S.L. problem $\frac{d^2y}{dx^2}+\lambda y=0$ with $y(0)=0,\ y(\pi)=0$. Further, discuss the convergence of this formal expansion. (10)

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PGIS-N-1020 B-17

M.A./M.Sc. Ist Semester Degree Examination

MATHEMATICS

(Algebra - I)

Paper : HCT - 1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

i. *Answer any **Five** questions.*

ii. *All questions carry equal marks.*

1. a) Define cyclic group let G be a cyclic group and H a subgroup of G. Then show that H is cyclic. **(8)**

   b) Let G be a finite cyclic group of order n. Then show that G has Ø (n) generations. **(8)**

2. a) Let $a \in G$ and $b \in G$ such that ab=ba, If O (a)=n and O (b)= m with m and n relatively prime then show that O (ab) = mn show that the group. **(8)**

   b) G is a direct product of subgroups H and K if and only if

      i) Every $x \in G$ can be uniquely expressed as x=hk, $h \in H, k \in K$

      ii) hk=kh, $h \in H, k \in K$. **(8)**

3. a) Let G be a group and $a \in G$ Then show that the numbers (*finite or infinite*) of elements in the conjugate class G (a) is equal to the index of the normaliser N(a) of a in G. **(8)**

   b) Let G be a finite group of order n and p a prime with $p^k \mid n$ and $p^{k+1} \nmid n$ then prove that G has a subgroup of order p^k. **(8)**

4. a) Let G be a finite group then prove that.

   i) any two sylow P-subgroups are conjugate to each other.

   ii) The numbers of sylow p-subgroups is of the form $1+tp, t\geq 0$ **(8)**

   b) Define solvable group prove that a group G is solvable iff $G^{(n)}=\{e\}$ for some $n\geq 1$. **(8)**

5. Prove that every integral domain can be embedded in a field. **(8)**

6. a) State and prove FERMAT theorem. **(8)**

   b) If R is a commutative ring with unit element so is R [x]. If R is an integral domain then prove that R [x] is also an integral domain. **(8)**

7. a) If F is a field then prove that F [x] is a Euclidean domain. **(8)**

   b) Let R he a WFD then prove that the polynomial ring R [x] is also a WFD. **(8)**

8. a) lET K|F and L|K be algebraic extensions then prove that L|F is an algebraic extension. **(8)**

   b) Let Gh F = P > O. Then show that F is perfect if and only is every element of F has a p^{th} root in F. **(8)**

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Roll No. ______________________ [Total No. of Pages : 2

PGIS-O 1019 B-17
M.A/M.Sc. Ist Semester Degree Examination
MATHEMATICS
(Real Analysis)
Paper : HCT - 1.1
(CBCS Old Scheme)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) *Answer any five questions.*
2) *All questions carry equal marks.*

1. a) If P* is a refinement of P then prove that $L(p,f,\alpha)\le L(p^*,f,\alpha)$ and $U(p^*,f,\alpha)\le U(p,f,\alpha)$ **(8)**

b) State and prove first mean value theorem. **(8)**

2. a) If f is monotonic on [a,b] and α is continuous on [a,b] then show that $f\in\mathbb{R}(\alpha)$. **(8)**

b) State and prove second - mean value theorem. **(8)**

3. a) Define meaning of functions of bounded variation. Show that the product of two functions of bounded variation is also of bounded variation. **(8)**

b) Assume α increases monotonically and $\alpha'\in R$ on [a,b]. Let f be a bounded real function on [a,b]. Then prove that $f\in\mathbb{R}(\alpha)$ if and only, if $f\alpha'\in R$. Also prove that $\int_a^b f d\alpha=\int_a^b f(x)\alpha'(x)dx$. **(8)**

4. a) If f maps [a,b] into R and if $f\in\mathbb{R}(\alpha)$ for some monotonically increasing function α on [a,b], then prove that $|f|\in\mathbb{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right|\le\int_a^b |f| d\alpha$ **(8)**

b) Show that there exists a continuous function on the real line which is nowhere differentiable. **(8)**

5. State and prove stone -Weier strass theorem. **(16)**

6. a) Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dim $X \leq r$. **(8)**

   b) If $A \in L(R^n, R^m)$, then show that $\|A\| < \infty$ and A is uniformly continuous mapping of R^n into R^m. if $A, B \in L(R^n, R^m)$ and C is a selar then show that $\|A+B\| \leq \|A\| + \|B\|$, $\|CA\| = |C|\|A\|$. If $\|A-B\|$ is a distance between A and B then show that $L(R^n, R^m)$ is a metric space. **(8)**

7. a) Suppose f maps a convex open set $E \subset R^n$ in to R^m, f is differentiable in E, there is a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then show that $|f(b) - f(a)| \leq M|b-a|$ for all $a \in E, b \in E$. **(8)**

   b) If a function f be such that $(n-1)^{th}$ derivative f^{n-1} is continuous in [a, a+h] and its derivative f^n exists in [a,a+h]. Prove that the functions $f, f', f'', ... f^{n-1}$ exists and are continuous in [a, a+h] while f^n exists in [a, a+h] **(8)**

8. State and prove implicit function theorem. **(16)**

Roll No. ______________________ [Total No. of Pages : 3

PGIS-N-1018 B-17
M.A./M.Sc. Ist Semester (CBCS) Degree Examination
MATHEMATICS
(Real Analysis)
Paper : HCT 1.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to the Candidates :

i) *Answer any **Five** questions.*

ii) *All questions carry **equal** marks.*

1. a) If P* is a refinement of P, then show that $L(p^*, f, \alpha) \geq L(p, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(p, f, a)$. **(8)**

b) If $f \in \mathbb{R}(\alpha_1)$ and $f \in \mathbb{R}(\alpha_2)$ Then prove that $f \in \mathbb{R}(\alpha_1 + \alpha_2)$ and $\int_a^b f d(\alpha_1 + \alpha_2) = \int_a^b f d\alpha_1 + \int_a^b f d\alpha_2$. **(8)**

2. a) State and prove second mean value theorem. **(8)**

b) If $f \in \mathbb{R}$ on [a, b], are α is monotone increasing on [a,b] such that $\alpha^1 \in \mathbb{R}$ on (a,b) then prove that $f \in \mathbb{R}(\alpha)$ and $\int_a^b f d\alpha = \int_a^b f \alpha^1 dx$. **(8)**

3. a) If a sequence $\{f_n\}$ converges uniformly in [a, b] and x_0 is a point of [a, b] such that $\lim fn(x) = an$, n = 1.2,.... them prove that $n \to \infty$: **(8)**

i) < an > converges

ii) $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to \infty} an$

b) Show that of a series Σfn converges uniformly to f in [a, b] and its terms fn are continuous at a point $x_0 \in [a,b]$ then the sum function f is also continuous at x_0 **(8)**

4. a) Prove that if a sequence <fn> of continuous function defined on [a, b] is monotonic increasing and converges to a continuous function f then the convergence is uniform on [a, b]. **(8)**

b) If K is a compact metric space, if $fn \in \ell(k)$ for n = 1, 2,.... and if {fn} converges uniformly on K, then show that {fn} is equicontinuous on K. **(8)**

5. State and prove Stone-Weierstrass theorem. **(16)**

6. a) If a power series $\sum_{n=0}^{\infty} anx^n$ converges at the end point $x = R$ of the interval of convergence]-R, R [, then prove that it is uniformly convergent in the closed interval [O, R]. **(8)**

b) Show that if a function f is bounded and integrable in [o, a], a > o, and monotone in] o, δ] o ≤ δ < a, then $\lim \int_0^a f \frac{\sin nx}{x} dx = f(0+) \int_0^\infty \frac{\sin x}{x} dx$ **(8)**

7. a) Show that a linear operator T on a finite dimensional vector space X is one-to-one if and only if the range of T is all of X. **(8)**

b) Prove the following : **(8)**

i) If $T \in L(R^n, R^m)$ then $\| T \| < \infty$ and T is a uniformly continuous mapping of R^n into R^m.

ii) If $T, S \in L(R^n, R^m)$ and c is a solar, then $\|T+S\| \leq \|T\| + \|S\|$, $\|CT\| = |C| \|T\|$ with the distance between T and S defined as $\|T-S\|$, $L(R^n, R^m)$ is a metric space

iii) If $T \in L(R^n, R^m)$ and $S \in L(R^m, R^k)$ then $\|ST\| \leq \|S\| \|T\|$.

8. a) Define functions of class. Show that Let f be a differentiable function whose domain D is open, connected set such that $df(x) = 0 \forall x \in D$. Then f is a constant function.

(8)

b) Define contraction principle. Prove that if X is a complete metric space and if ϕ is a contraction of X into X then there exists one and only one $x \in X$ such that $\phi(x) = x$. If $\phi(x) = x$ and $\phi(y) = y$ then $d(\phi(x)\, \phi(y) \leq cd(x, y)$ gives $d(x, y) \leq cd(x, y)$ happens only when $d(x, y) = 0$. **(8)**