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PGIIIS-N 1528 B-2K13

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Functional Analysis)
Paper - HCT-3.1
(New)

Time : 3 Hours Maximum Marks :80

Instructions to Candidates:

i) Answer any **five** questions

ii) All questions Carry **equal** marks.

1. a) Define a metric space. If (x,d) is a metric space, then prove the following **(8)**

   i) The union of any number of open sets in x is open

   ii) The intersection of a finite number of open sets in x is open.

   b) State and prove cantor's intersection theorem. **(8)**

2. a) Prove that a metric space is sequentially compact iff it has the Bolzano weirstrass property. **(8)**

   b) State and prove Lebesgue Covering Lemma. **(8)**

3. a) Define a Banach space. If $B(X,R)=\{f/f:X\rightarrow R \text{ is bounded }\}$, where $X\neq\phi$ and norm of f is defined by $\|f\|=\underset{x\in x}{Sup}\{f(x)/f\in B(X,R)\}$ then show that B(X,R) is a Banach space **(8)**

   b) If X and Y are two normed linear spaces, then prove that X is topologically iso-morphic to Y iff $\exists$ a linear map T:X$\rightarrow$Y which is onto and $\exists$ a and positive constants m and M such that $m\|x\|\leq\|T(x)\|\leq M\|x\|, \forall x\in X$. **(8)**

4. a) State and prove Riesz's Lemma. **(8)**

   b) State and prove Banach-Steinhans theorem **(8)**

5. a) State and prove Hahn-Banach theorem. (8)

b) State and Prove open mapping theorem (8)

6. a) Let N be an arbitrary normed linear space. Show that for each vector x in N, the scalar valued function F_x defined by $F_x(f) = f(x), \forall f \in N^*$ is a continuous linear functional and the mapping $\psi : x \to F_x$ is an isometric isomorphism of N into N^{**}. (8)

b) Define a Hilbert space. State and prove cauchy schwartz's inequality. (8)

7. a) Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. (8)

b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then show that the linear subspace M+N is also closed. (8)

8. a) State and prove Riesz representation theorem. (8)

b) Show that the adjoint of T is a unique continuous linear operator on H with the definition $(Tx, y) = (x, T^* y)$ (8)

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PGIIIS-N 1529 B-2K13

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination

Mathematics

(Graph Theory) - I

Paper - HCT-3.2

(New)

Time : 3 Hours Maximum Marks :80

Instructions to Candidates:

i) Answer any **five** fullquestions

ii) All questions Carry **equal** marks.

1. a) Prove that the number of vertices of odd degree in a graph is always even. **(5)**

b) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree2. Draw the two such graphs. **(6)**

c) In any connected graph, define the following terms:

i) Walk ii) Path iii) Trail

iv) Circuit v) Cycle. **(5)**

2. a) Define cut vertex in a connected graph. Show that a vertex 'v' of a connected graph G is a cut vertex of G if and only if there exist vertices u and w $(u, w \neq v)$ such that v is on every u-w path of G. **(6)**

b) Construct cubic graphs with 6 and 8 vertices. **(4)**

c) Show that for any graph G with six vertices G or $\overline{G}$ contains a cycle. Also illustrate through an example. **(6)**

3. a) Define a tree. Show that a (p,q) graph is a tree if and only if it is acyclic and $p = q + 1$ **(6)**

b) Define a spanning tree in a connected graph. Show that every connected graph contains a spanning tree. **(5)**

c) Show that a connected graph G is a tree if and only if every edge is a bridge. **(5)**

4. a) Define eccentricity, radius and diameter in a graph. Show that every tree has center consisting of either one vertex or two adjacent vertices. **(8)**

b) Define Rank and Nullity in a spanning tree. Prove that a graph is a tree if and only if it is minimally connected **(8)**

5. a) Define vertex and edge connectivity of a graph with an example. Prove that in any graph. $K(G) \leq \lambda(G) \leq \delta(G)$. **(12)**

b) State the Graphical variations of Menger's theorem. **(4)**

6. a) Let G be a nontrivial connected graph. Then prove that G contains an Eulerian trail if and only if G has exactly two odd vertices. **(8)**

b) Find under what condition the complete bipartite graph $K_{m,n}$ has an Eulerian graph. **(8)**

7. a) Let G be a graph with $p \geq 3$ vertices and $\delta \geq p/2$ then show that G is Hamiltonian. **(8)**

b) Prove that a graph H is the block graph of some graph if and only if every block of H is complete. **(8)**

8. a) Show that a graph is the line graph of a tree if and only if it is a connected block graph in which each cut vertex is on exactly two blocks. **(8)**

b) Prove that every tournament has a spanning path. Explain with an example. **(8)**

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PGIIIS-N 1530 B-2K13

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Computational Numerical Methods-I)
Paper - HCT-3.3
(New)

Time : 3 Hours Maximum Marks :80

Instructions to Candidates:

i) Answer any **five** questions

ii) All questions Carry **equal** marks.

1. a) Derive the Hermite interpolating polynomial. **(8)**

b) Construct the Hermite interpolation polynomial that fits the data.

x	0	0.5	1.0
$f(x)$	0	0.4794	0.8415
$f'(x)$	1	0.8776	0.5403

(8)

Estimate the value of f(0.25) and f(0.75)

2. a) Obtain the piecewise quadratic interpolating polynomial for the function f(x) defined by the data

x	-3	-2	-1	1	3	6	7
$f(x)$	369	222	171	165	207	990	1779

Hence find an approximate value of $f(-2.5)$ and $f(6.5)$ **(8)**

b) Using the chebyshev polynomials, obtain the least squares approximation of second degree for $f(x)=x^4$ on [-1,1]. **(8)**

3. a) Describe Newton -Raphson method for solving the system of two non-linear equations with two unknowns. **(8)**

b) Apply Newton-Raphson method for solving $x^2+y^2-4=0$

$$xy-1=0$$

with $(x_0, y_0) = (2,0)$. Perform two iterations. **(8)**

4. a) Describe Bairstow method to extract a quadratic factor (x^2+px+q) from the polynomial $P_n(x) = a_0x^n + a_1x^{n-1} + \dots\dots + a_{n-1}x + a_n$ and $a_0 \neq 0$ **(8)**

b) Perform two iterations of Bairstow method to extract a quadratic factor $(x^2 + px + q)$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use the initial approximation $p_0 = -0.9$ and $q_0 = 0.9$. **(8)**

5. a) Describe Gauss-elimination method for solving system of three equations with three unknowns. **(8)**

b) Solve the system of equations

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

by Gauss-Jordan method **(8)**

6. a) Describe Jacobi's iteration method for solving the system of n-linear equations with n-unknowns. **(8)**

b) Solve the following system of equations by using Jacobi's iteration method.

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Take the initial approximation as $X^{(0)} = [0.5, -0.5, -0.5]^T$ and perform three iterations. **(8)**

7. a) Describe Gauss-Jordan method of matrix inversion **(8)**

b) Find the inverse of the following matrix by Jordan method. $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$ **(8)**

8. a) Describe Jacobi's method to find all the eigen values and eigen vectors of a real symmetric matrix. **(8)**

b) Find the eigen values and the eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 2 \end{bmatrix}$ by Givans method. **(8)**

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PGIIIS-N 1532 B-2K13
M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Operations Research - II)
Paper - OET 3.1
(New)

Time :3 Hours Maximum Marks : 80

Instructions to candidates:

1) Answer any **five** questions.
2) All questions carry **equal** marks.

1. a) Explain two - phase method of solving L.P.P. **(8)**

b) Use two-phase method to solve the following L.P.P.

$$\begin{aligned} &\textit{Maximize } Z = 5x_1 - 4x_2 + 3x_3 \\ &\textit{Subject to } 2x_1 + x_2 - 6x_3 = 20 \\ &\qquad 6x_1 + 5x_2 + 10x_3 \le 76 \\ &\qquad 8x_1 - 3x_2 + 6x_3 \le 50 \\ &\textit{and} \quad x_1, x_2, x_3 \ge 0 \end{aligned}$$

(8)

2. a) Write the dual of the following L.P.P. **(6)**

$$\begin{aligned} &\textit{Minimize } Z = 2x_2 + 5x_3 \\ &\textit{Subject to } x_1 + x_2 \ge 2 \\ &\qquad 2x_1 + x_2 + 6x_3 \le 6 \\ &\qquad x_1 - x_2 + 3x_3 = 4 \\ &\textit{and } x_1, x_2, x_3 \ge 0 \end{aligned}$$

b) Prove that dual of the dual is primal. **(10)**

3. a) Explain

i) payoff matrix

ii) The Maximin Minimax principle **(8)**

b) solve the following game and determine the value of the game $A\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$ **(8)**

4. a) solve the following 2×3 game graphically (8)

Player B

$$\text{Player A}\quad \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

b) Using the principle of dominance, solve the following game

Player B

$$\text{Player A}\quad \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$ (8)

5. a) Explain a queueing system with its characteristics. (8)

b) Explain arrival process. (8)

6. a) Discuss (M/M/1): (GD/N/∞) model. (8)

b) In a car wash facility, information gathered indicates that cars arrive for service to a Poisson distribution with mean 5 per hour. The time for washing and cleaning each car varies but is found to follow an exponential distribution. With mean 10 minutes per car. Suppose that the facility has a total of 5 parking spaces. If the parking lot is full, newly arriving cars back to seek service elsewhere. Obtain how many customers are back due to the limited parking space, compute the expected waiting time until a car is washed. (8)

7. a) Explain the concept of event type simulation. (8)

b) Customers arrive at a milk booth for the required service. Assume that inter arrival and service time are constants and given by 1.8 and 4 time units respectively. Simulate the system by hand computation for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility? (8)

8. a) Illustrate how the random observations can be computed for

i) Exponential distribution.

ii) Erlang distribution.

iii) Poisson distribution. (8)

b) Write the steps in simulation. (8)

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PGIIIS-N 1531 B-2K13

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Fluid Mechanics-I)
Paper - SCT-3.1
(New)

Time : 3 Hours Maximum Marks :80

Instructions to Candidates:

i) Answer any **five** questions

ii) All questions Carry **equal** marks.

1. a) Explain about Lagrange and Euler's method in Fluid motion. Derive $\frac{\partial \varrho}{\partial t}+\nabla\left(\varrho\bar{g}\right)=0$ **(8)**

   b) Find the path lines and streak lines for the velocity $\bar{q}=\left(\frac{x}{t},y,0\right)$. **(8)**

2. a) Derive Lamb's Hydrodynamical equation. **(8)**

   b) Show that the difference of the values of stream function at the two points represents the flux of a fluid across any curve joining the two points. **(8)**

3. a) Define sources, sinks. Find the complex potential for a doublet. **(8)**

   b) Derive the complex potential for the image of a doublet relative to a circle. Define confirmal transformation. **(8)**

4. a) Fluid is coming out from a small hole of cross-section σ_1 in a tank, if the minimum cross-section of the stream coming out of the hole is σ_2, then show that $\frac{\sigma_2}{\sigma_1}=\frac{1}{2}$ **(8)**

   b) Prove that for liquid circulating irrotationally in part of the plane between two non-intersecting circles, the curves of constant velocity are Cassini's ovals. **(8)**

5. a) Discuss the motion of a circular cylinder moving with velocity U along x-axis in an infinite mass of liquid at rest at infinity. **(8)**

b) State and prove Blasius theorem. **(8)**

6. a) A circular cylinder is fixed across stream of velocity U with circulation K around the cylinder. Show that the maximum velocity in the liquid is $2U+\frac{K}{2\pi a}$, where a is the radius of the cylinder. **(8)**

b) Show that the velocity potential of sphere is $\phi=\left[Ar^{n}+Br^{-(n+1)}\right]P_n(\mu)$ where $\mu=\cos\theta$ and (r,θ,ω) are the spherical co-ordinates. **(8)**

7. a) Liquid is contained in a rotating elliptic cylinder. By making use of elliptic transformation $Z=C\cosh and$, show that the stream function of the motion is

$$\psi=\frac{1}{2}\omega\left(\frac{a^2-b^2}{a^2+b^2}\right)\left(x^2-y^2\right).$$ **(8)**

b) Show that when an infinitely long cylinder of density σ whose cross section is ellipse of semi axes a; b is immersed in an infinite liquid of density ϱ, the square of its radius of gyration about its axis is effectively increased by the quantity

$$\frac{\varrho}{8\sigma}\frac{\left(a^2-b^2\right)^2}{ab}$$ **(8)**

8. a) Find the necessary and sufficient condition that vortex lines may be at right angles to the stream lines. **(8)**

b) Derive vorticity transport equation. **(8)**

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