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PGIIIS-N-1533 B-18

M.A/M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Operations Research - II

Paper - OET 3.1 (New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five questions.*
2) *All questions carry equal marks.*

1. a) Use Big-M method to solve the following L.P.P. **(8)**

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

and $x_1, x_2, x_3\ x_4 \geq 0$

b) Use two-phase simplex method to solve the following L.P.P. **(8)**

Maximize $z = x_1 + 2x_2 + 3x_3$

Subject to $x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_1 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$

2. a) Prove that the dual of the dual is primal. **(8)**

b) Use dual simplex method to solve the following L.P.P. **(8)**

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

and $x_1, x_2 \geq 0$

3. a) Explain : (8)

i) Payoff matrix

ii) Maximin-Minimax principle

b) Solve the game whose payoff matrix is (8)

Player B

$$\text{Player A} \begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix}$$

4. a) Solve the following game graphically (8)

Player B

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \end{array}$$

b) Using the principle of dominance, solve the following game. (8)

Player B

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 9 & 2 \\ 8 & 6 \\ 6 & 4 \end{bmatrix} \end{array}$$

5. a) Define a queuing system and explain in brief the characteristics of the queuing system. **(8)**

b) Explain departure process. **(8)**

6. a) Discuss (M/M/1) : (N/FIFO) model. **(8)**

b) Discuss (M/M/1) : (∞/FIFO) model. **(8)**

7. a) Write steps in simulation. **(8)**

b) Discuss event-type simulation. **(8)**

8. a) Write the procedure of the monte-carlo simulation. **(8)**

b) Discuss simulation languages. **(8)**

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PGIIIS-O-1533 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Operations Research - II

Paper - OET 3.1 (Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five questions.*
2) *All questions carry equal marks.*

1. a) Explain Big-M method for solving L.P.P. **(8)**

b) Use two-phase simplex method to solve the following L.P.P. **(8)**

Minimize $z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$

2. a) Prove that dual of the dual is primal. **(8)**

b) Solve the following L.P.P. by dual simplex method **(8)**

Minimize $z = 10x_1 + 6x_2 + 2x_3$

Subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$

3. a) Explain : **(8)**

i) Two-person zero-sum games

ii) Pay-off matrix

b) Determine which of the following two-person zero-sum games are strictly determinable and fair. Give optimum strategies for each player in the case of strictly determinable games. **(8)**

i) Player B

Player A $\begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix}$

ii) Player B

Player A $\begin{bmatrix} 10 & 6 \\ 8 & 2 \end{bmatrix}$

4. a) Solve the following game graphically **(8)**

Player B

Player A $\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$

b) Using the principle of dominance, solve the following game **(8)**

Player B

Player A		B_1	B_2	B_3	B_4	B_5
	A_1	3	5	4	9	6
	A_2	5	6	3	7	8
	A_3	8	7	9	8	7
	A_4	4	2	8	5	3

5. a) Explain a queueing system with its characteristics. **(8)**

b) Explain arrival process. **(8)**

6. a) Discuss (M/M/1) : (N/FIFO) model. **(8)**

b) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Also, find the average waiting time of a new train coming into the yard. **(8)**

7. a) Write steps in simulation. (8)

b) Discuss event-type simulation. (8)

8. a) Write the procedure of the Monte-Carlo simulation. (8)

b) Customers arrive at a milk booth for the required service. Assume that inter-arrival and service times are constant and given by 1.8 and 4 time units respectively. Simulate the system by hand computations for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility. (8)

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PGIIIS-N-1529 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

(Functional Analysis)

Paper - HCT 3.1 (New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five questions.*
2) *All questions carry equal marks.*

1. a) Define a metric space If (X, d) is a metric space, then prove the following. **(8)**

   i) The union of any number of open sets in X is open.

   ii) The intersection of a finite number of open sets in X is open.

   b) State and prove Baire's category theorem. **(8)**

2. a) If X is a complete metric space and Y is a subspace of X, then prove that Y is complete if and only if it is closed. **(8)**

   b) State and prove Cantor's intersection theorem. **(8)**

3. a) Show that R^n is a Banach space with norm defined by **(8)**

$$\|x\| = \|(x_1, x_2, \ldots, x_n)\|$$

$$= \left[\sum_{i=1}^{n} |x_i|^2\right]^{1/2}$$

   b) State and prove Riesz's lemma. **(8)**

4. a) State and prove Hahn-Banch theorem. **(8)**

   b) State and prove open mapping theorem. **(8)**

5. a) If B and B' are Banach spaces and if T is continuous linear transformation of B onto B', then show that T is an open mapping. **(8)**

b) If M is a linear subspace of a normed linear space N and if f is a functional defined on M, then show that f can be extended to a functional f_0 on the whole space N such that $f_0(x) = f(x), \forall x \in M$ & $\|f_0\| = \|f\|$. **(8)**

6. a) Define a Hilbert space. Further state and prove Cauchy schwartz's inequality. **(8)**

b) If M and N are closed linear subspace of a Hilbert space H such that $M \perp N$, then show that the linear subspace $M + N$ is also closed. **(8)**

7. a) State and prove Riesz representation theorem. **(8)**

b) State and prove projection theorem. **(8)**

8. a) Define a self adjoint operator on a Hilbert space H. Further show that an operator T on H is self adjoint if and only if (Tx, x) is a real for all x. **(8)**

b) State and prove spectral theorem. **(8)**

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PGIIIS-O-1532 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper - SCT 3.1 (Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) Answer any five questions.

2) All questions carry equal marks.

1. a) Derive equation of continuity by Lagrangian and Eulerian method. **(8)**

   b) Find the stream lines and paths of the particles for the two dimensional velocity field

   $u = \frac{x}{1+t}, \; v = y, \; w = 0$. **(8)**

2. a) Show that the variable ellipsoid $\frac{x^2}{a^2k^2t} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form for the boundary surface of a liquid at any time t. **(8)**

   b) Derive equations for impulsive actions. **(8)**

3. a) Define sources and sinks. Find the complex potential for a doublet. **(8)**

   b) Prove that the difference of the values of stream function at the two points represents the flux of a fluid across any curve joining the two points. **(8)**

4. a) Derive the image of a circle with respect to a circle of radius a. **(8)**

   b) State and prove Milne thomson circle theorem. **(8)**

5. a) Discuss the motion of a circular cylinder moving with velocity U along x-axis in an infinite mass of liquid at rest at infinity. **(8)**

   b) Find the velocity potential and stream function at any point of a liquid contained between coaxial cylinders of radii a and b ($a < b$) when the cylinders are moved suddenly parallel to themselves in directions at right angles with velocities u and v respectively **(8)**

6. a) State and prove Blasius theorem. **(8)**

b) Show that the velocity potential of sphere is $\phi = [A r^n + B r^{-(n+1)}] p_n(\mu)$ where $\mu = \cos\theta$ and (r, θ, w) are the spherical coordinates. **(8)**

7. a) Show that the fluid pressures exerts a force $\frac{M\dot{U}}{2}$ opposing the motion. **(8)**

b) Determine the stagnation points for $\frac{\partial \phi}{\partial r} = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta$. **(8)**

8. a) Derive the necessary and sufficient condition that vortex lines may be at right angles to the stream lines. **(8)**

b) Derive vorticity transport equation. **(8)**

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PGIIIS-N-1532 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Fluid Mechanics - I

Paper - SCT 3.1 (New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five questions.*
2) *All questions carry equal marks.*

1. a) Derive the equation for local and individual time rate of change. Define stream line, path line and streak lines. (8)

   b) Show that surfaces exist which cut stream lines orthogonally if the velocity potential exists. (8)

2. a) Define boundary surface. Derive boundary conditions on velocity and on pressure. (8)

   b) Find the stream lines and paths of the particles for the two dimensional velocity field

   $u = \frac{x}{1+t},\ v = y,\ w = 0.$ (8)

3. a) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two dimension fluid. Show that the stream lines at time t are the curves $(x-t)^2 - (y-t)^2$ = constant and that the paths of fluid particles have the equations

   $\log(x-y) = \frac{1}{2}\left\{(x+y) - a(x-y)^{-1}\right\} + b$ where a and b are constants. (8)

   b) State and prove Bernoullis equation. (8)

4. a) State and prove kelvins circulation theorem. (8)

   b) What arrangements of sources and sinks will give rise to the function $w = \log\left(z - \frac{a^2}{z}\right)$

   Draw a rough sketch of the stream lines in this case and prove that two of them sub-divide into the circle $r = a$ and axis of y. (8)

5. a) Discuss the motion of a circular cylinder moving with velocity U along x-axis in an infinite mass of liquid at rest at infinity. **(8)**

b) Find the kinetic energy of an infinite mass of liquid moving irrotationally. **(8)**

6. a) An infinite cylinder of radius a and density σ is surrounded by a fixed concentric cylinder of radius b and the intervening space is filled with liquid of density ρ. Prove that the impulse per unit length necessary to start the inner cylinder with velocity V is

$$\frac{\pi a^2}{b^2 - a^2}\left[(\sigma+\rho)b^2 - (\sigma-\rho)a^2\right]V.$$ **(8)**

b) In case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u, in a fixed direction where u is a variable. Show that the maximum value of velocity at any point of the fluid is $2u$. Prove that the necessary pressure to hold the disc at rest is $2mu$ where m is the mass of liquid, displaced by the disc. **(8)**

7. a) Prove that the necessary and sufficient condition for irrotational motion is that there exists a potential ϕ such that $\vec{q} = -\Delta\phi, q$ being velocity vector. **(8)**

b) State and prove kelvins minimum energy theorem. **(8)**

8. a) Prove that if $\lambda = \frac{\partial u}{\partial t} - \gamma\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + w\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$ and μ, γ have two similar expressions, then $\lambda dx + \mu dy + \gamma dz$ is a perfect differential if the forces all conservative and density is constant. **(8)**

b) Liquid of density ρ is following in two dimensions between the oval curves $r_1 r_2 = a^2$, $r_1 r_2 = b^2$ where r_1, r_2 are the distances measured from two fixed points, if the motion is irrotational and quantity q per unit time crosses any line joining the bounding curves, then the kinetic energy is $\frac{\pi \rho q^2}{\log(b|a)}$. **(8)**

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PGIIIS-N-1530 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Graph Theory - I

Paper - HCT 3.2 (New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five full questions.*
2) *All questions carry equal marks.*

1. a) Define a regular graph. Construct cubic graphs with 6 and 8 vertices. **(6)**

b) Prove that every self-complementary graph has 4 n or 4n+1 vertices. **(5)**

c) Define product and composition operations. Draw the following graphs: **(5)**

i) $K_3 \times K_3$ ii) $K_{1,3} \times K_{1,3}$

iii) $P_2[P_3]$

2. a) Write STEPS for detection of planarity of a graph. Further show that K_5 and $K_{3,3}$ are nonplanar. **(8)**

b) Define maximal planar graph. If G is a maximal planar (p, q) graph with $p \geq 3$ then show that $q = 3p - 6$. **(8)**

3. a) Define self dual graph. For any connected plane graph G prove that $G \cong G^{**}$ with an example. **(8)**

b) Define crossing number and thickness of a graph. Determine $Cr(K_{1,2,2,2})$. **(8)**

4. a) For any graph G, show that $\chi(G) \leq 1 + \max \delta(G')$, where the maximum is taken over all induced subgraphs G' of G. Also find the chromatic number of **(8)**

i) $\overline{K_6}$ ii) $\overline{K_{m,n}}$ iii) $\overline{P_4}$

b) Prove that for any graph G, the sum and product of χ and $\overline{\chi}$ satisfy the inequalities,

$$2\sqrt{p} \leq \psi + \overline{\psi} \leq p+1, \; p \leq \psi\overline{\psi} \leq \left(\frac{p+1}{2}\right)^2.$$ **(8)**

[Contd....

5. a) Find the chromatic number of cubic graphs with $p = 8, 10$ vertices which does not contains a triangle and contains a triangle. Also write the steps for welsh and powell algorithm. **(8)**

b) Write the steps for the smallest last sequential algorithm with an example. **(8)**

6. a) Define an uniquely colorable graph. Show that every uniquely 4-colorable planar graph is maximal planar. **(8)**

b) For any nontrivial connected graph G, prove that $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$. **(8)**

7. a) Prove that every bridgeless cubic graph contains a 1-factor. **(8)**

b) Show that for every positive integer n, the grpah K_{2n+1} can be factored into n-Hamiltonian cycles. **(8)**

8. a) Define an arborescence of a diagraph. Prove that an arborescence is a tree in which every vertex other than the root has an indegree of exactly one. **(8)**

b) Prove that every tournament has a Hamiltonian path. **(8)**

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PGIIIS-O-1530 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Graph Theory - I

Paper - HCT 3.2 (Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) Answer any **five** full questions.
2) All questions carry **equal** marks.

1. a) Prove that every u–v walk contains a u–v path. **(5)**

b) Show that every simple graph must have atleast two vertices of the same degree. **(6)**

c) In any connected graph, define the following terms : **(5)**

i) Walk ii) Path

iii) Trail iv) Circuit

v) Cycle

2. a) Define a Bipartite graph. Show that a graph G is bipartite if and only if all its cycles are even. **(8)**

b) Construct cubic graphs with 6 and 8 vertices. **(4)**

c) Show that for any graph G with six vertices G or $\bar{G}$ contains a cycle. **(4)**

3. a) Show that a (p, q) graph G is a tree if and only if it is connected and $p = q + 1$. Also prove that a connected graph G is a tree if and only if every edge is a bridge. **(10)**

b) Show that every non trivial tree contains atleasst two end vertices. **(6)**

4. a) Define center in a tree. Show that every tree has either one or two centers. **(8)**

b) Show that in every network the value of a maximum flow equals the capacity of a minimum cut. **(8)**

5. a) For any graph G, prove that $K(G) \leq \lambda(G) \leq \delta(G)$ construct a graph with $k = 2, \lambda = 3, \delta = 4$. **(8)**

b) Let G be a nontrivial connected graph. Prove that G contains an Eulerian trail if and only if G has exactly two odd vertices. **(8)**

6. a) Explain the conditions for complete bipartite graph $k_{m,n}$ has an Eulerian graph. **(8)**

b) Is it possible to construct an Eulerian graph with odd number of vertices and even number of edges. Give an example. **(8)**

7. a) Let G be a graph with $p \geq 3$ vertices and $\delta \geq p/2$ then show that G is Hamiltonian. **(8)**

b) Draw the following graphs: **(8)**

i) An Eulerian graph which is not Hamiltonian.

ii) An Eulerian graph which is also Hamiltonian.

iii) A Hamiltonian graph which is not Eulerian.

iv) A graph which is neither Eulerian nor Hamiltonian.

8. a) Show that every tournament has a spanning path. **(8)**

b) Prove that every tournament has a Hamiltonian path. **(8)**

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PGIIIS-N-1531 B-18

M.A./M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Computational Numerical Methods - I

Paper - HCT 3.3 (New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any five full questions.*
2) *All questions carry equal marks.*

1. a) Describe Newton-Raphson method of solving system of two non-linear equations in two unknowns. **(8)**

   b) Perform two iterations of Newton-Raphson method to solve **(8)**

   $$x^2 + y^2 - 4 = 0$$
   $$xy - 1 = 0$$
   with $(x_0, y_0) = (2, 0)$.

2. a) Describe Graeffe's root squaring method to find the roots of the polynomial with real coefficients. **(8)**

   b) Find all the roots of the polynomial $x^3 - 8x^2 + 17x - 10 = 0$, by using Graeffe's root squaring method. **(8)**

3. a) Define an interpolating polynomial $p(x)$. Obtain the polynomial approximation $p(x)$ to $f(x) = e^{-x}$ using Taylor's expansion about $x_0 = 0$ and determine x when the error in $p(x)$ obtained from the first four terms only is to be less than 10^{-6} after rounding. **(8)**

   b) For the following data, calculate the differences and obtain the Gregory-Newton forward and backward difference interpolation polynomials and interpolate at $x = 0.25$ and $x = 0.35$. **(8)**

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

4. a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the following data. **(8)**

x	1	2	4	8
$f(x)$	3	7	21	73

b) For the following data fit quadratic splines with $M(0) = f''(0) = 0$ and hence find an estimate of $f(2.5)$. **(8)**

x	0	1	2	3
$f(x)$	1	2	33	244

5. a) Derive Lagrange bivariate interpolating polynomial. **(8)**

b) Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{1/2}$ on [0, 1]. **(8)**

6. a) Describe Gauss-Jordan elimination method of solving linear system of three equations with three unknowns. **(8)**

b) Solve the system of equations **(8)**

$$x + y + z = 1$$
$$4x + 3y - z = 6$$
$$3x + 5y + 3z = 4$$

by using Gauss-Jordan method.

7. a) Describe Jacobi's iteration method of solving linear system of equations. **(8)**

b) Solve the system of equations **(8)**

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

by using Jacobi's iteration method.

8. a) Solve the following system of equations. **(8)**

$$3x + 2y = 4.5$$
$$2x + 3y - z = 5.0$$
$$-y + 2z = -0.5$$

By SOR method. Find the optimum relaxation parameter and perform two iterations.

b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Cholesky's method. **(8)**
