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PGIS-N 1018 B-14

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Real Analysis)

Paper - HCT- 1.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

i) Answer any **five** questions

ii) All questions carry **equal** marks.

1. a) Define Riemann-steiltje's integrals and describe their existence. Prove that f is integrable with respect to α over [a,b] iff for every $\in > 0$ and for every partition P of [a,b] such that $U(P,f,\alpha) - L(P,f,\alpha) < E$ **(8)**

   b) Prove that if $f_1 \in \mathbb{R}(\alpha)$, $f_2 \in \mathbb{R}(\alpha)$, over [a,b] then $f_1 + f_2 \in \mathbb{R}(\alpha)$ and

   $$\int_a^b f_1 + f_2 \, d\alpha = \int_a^b f_1 \, d\alpha \int_a^b f_2 \, d\alpha$$ **(8)**

2. a) Define integral as a limit of a sum. If $\lim S(p,f,\alpha)$ exists as $\mu(p) \to 0$ then prove that $F \in \mathbb{R}(\alpha)$ and $\lim_{\mu(p) \to 0} S(p,f,\alpha) = \int_a^b f \, d\alpha$ **(8)**

   b) State and prove the first mean value theorem. **(8)**

3. a) Define the meaning of functions of bounded variation. Show that the product of two functions of bounded variation is also of bounded variation. **(8)**

   b) If $F \in \mathbb{R}$ on [a,b] and $F(x) = \int_a^x f(t) dt$ for $a \le x \le b$ then prove that F is continuous on [a,b]. Also prove that if f is continuous at a point x_0 of [a,b] then F is differentiable at x_0 and $F^1(x_0) = f(x_0)$ **(8)**

4. a) Prove that a sequence of functions $\{f_n\}$ defined on [a,b] converges uniformly on [a,b] if and only if for every $\in > 0$ and for all $x \in [a,b]$ there exists an integer N such that $|f_n + p(x) - f_n(x)| < \in \forall \; n \ge N, P \ge 1$ **(8)**

b) If a series $\sum_{n=1}^{\infty} f_n$ converges uniformly to f in [a,b] and x_0 is a point in [a,b] such that

$$\lim_{x \to x_0} f_n(x) = a_n, n = 1,2,3,...$$

then prove that

i) $\sum_{n=1}^{\infty} a_n$ converges

ii) $\lim_{x \to x_0} f(x) = \sum_{n=1}^{\infty} an$ **(8)**

5. a) Define uniform convergence in a series. Show that the series

$$x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \frac{x^4}{(1+x^4)^3} + ...$$ **(8)**

b) If $\{f_n\}$ be a sequence of functions, differentiable on [a,b] and $\{f_n(x_0)\}$ converges at $x_0 \in [a,b]$. If $\{f_n'\}$ converges uniformly on [a,b] to f' then prove that $\{f_n\}$ converges uniformly on [a,b] to a function f and

$$f'(x) = \lim_{n \to \infty} f'n(x), \ a \le x \le b$$ **(8)**

6. State and the stone-Weierstrass theorem. **(16)**

7. a) Prove the followings:

i) If $T \in L(R^n, R^m)$, then $\|T\| < \infty$ and T is a uniformly continuous mapping of R^n into R^m

ii) If $T, S \in L(R^n, R^m)$ and C is a scalar, the $\|T+S\| \le \|T\| + \|S\|, \|CT\| = |C| \|T\|$

iii) If $T \in L(R^n, R^m)$, and $S \in L(R^m, R^k)$ then $\|ST\| \le \|S\| \|T\|$ **(8)**

b) If a function f be such that (n-1)th derivative f^{n-1} is continuous in *[a,a+h]* and its derivative f^n exists in *[a,a+h]*. Prove that the functions $f, f', f'', ..., f^{n-1}$ exists and are continuous in *[a,a+h]* while f^n exists in *[a,a+h]* **(8)**

8. State and prove the inverse function theorem. **(16)**

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PGIS - N 1026 B - 14
M.A./M.Sc. Ist Semester Degree Examination
Mathematics
(Classical Mechanics)
Paper - SCT 1.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) Answer any five questions
2) All questions Carry equal Marks

1. a) Deduce Lagrange's equation from D'Alemberts principle. **10**

b) Construct Lagragian and the equations of motion of a coplanar double pendulum placed in a uniform gravitational field **6**

2. a) Obtain hamilton's equation of a motion of a particle in a plane referred to a moving axes, where the components of velocities are $u = x - \omega y, v = y + \omega x$ **8**

b) Find Euler's dynamical equations of the motion of a rigid body about a fixed point on the about a fixed point on the body **8**

3. a) Show that

$$M_z = \frac{\partial L}{\partial \dot{\phi}_i} \text{ for a system with } L = \frac{1}{2} m_i \left(\dot{r}_i^2 + r_i^2 \dot{\phi}_i^2 + \dot{z}_i^2 \right) \text{-v(r)}$$ **8**

b) A Symmetrical top can turn freely about a fixed point in its axis of symmetry and is acted on by forces derived from the potential function $\mu Cot^2\theta, \theta$ is the angle between this axis and a fixed line, say OZ. show that the equation of motion can be integrated in terms of elementary functions. **8**

4. a) State hamilton's principle of least action and derive hamilton's canonical equation from hamilton's principle. **8**

b) Derive poincare integral invariant **8**

5. a) State Lee Haw -Chung theorem Obtain Jacobis equations by using Whittakeris equation. 8

b) Obtain hamiltun - jacobi equations for simple harmonic motion and find a complete integral and determine solution of it. 8

6. a) Show that if F,G are both integrals of motion, then so is their poisson bracket 8

b) Prove that the transformation given by $q = \sqrt{2P}\, SinQ$, $p = \sqrt{2P}\, CosQ$ is canonical by using poisson brackets. 8

7. a) Find the value of q and p for a harmonic oscillator described by hamilton

$H = \frac{1}{2}(p^2 + \omega^2 q^2)$ And generated by the function $F = \frac{1}{2}\omega q^2 Cot 2\pi Q$ 8

b) Show that Langrange's brackets is invariant under canonical transformation 8

8. a) Show that langrange's bracket donot obey the commutative law. Also prove that

i) $\{q_p, q_j\} = 0$

ii) $\{p_i, p_j\} = 0$

iii) $\{q_i, p_j\} = \delta_{ij}$ 8

b) Establish the relationship between Lagrange's and Poisson's bracket 8

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PGIS-N 1020 B-14

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Algebra-I)

Paper - HCT-1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

i) Answer any **five** questions

ii) All questions carry **equal** marks.

1. a) Let G be a finite cyclic group of order n. Then show that G has a unique subgroup of order d for every divisor d of n. **(8)**

   b) Show that S_n is a finite group of order $\lfloor n$ and is non-abelian if n>2. **(8)**

2. a) Let G be a finite group of order n. Then show that G is isomorphic to a subgroup of S_n **(8)**

   b) Let G be a group and $a \in$ G. Then show that the number of elements in the conjugate class G(a) is equal to the index of the normaliser N(a) of a in G **(8)**

3. a) Let G be a finite group of order Pq, P and q being distinct primes, and P<q. Then show that

   i) G has a unique sylow q-Subgroup

   ii) If P does not divide q-1, G has a unique Sylow' P-Subgroup, and then G is cyclic of order Pq. **(8)**

   b) Define solvable group. Show that

   i) any subgroup of a solvable group is solvable

   ii) any quotient group of a solvable group is solvable. **(8)**

4. Prove that every integral domain can be embedded in a field **(16)**

5. a) State and prove unique factorisation theorem **(8)**

   b) If R is a commutative ring with unit element, then show R[x] is also a commutative ring. If R is an integral domain show that R[x] is also an integral domain. **(8)**

6. a) Let R be a unique factorisation domain and F the quotient field of R. Let $f(x) \in R(x)$ be irreducible in $R[x]$. Then show that $f(x)$ is also irreducible in $F[x]$ (8)

b) State and prove first isomorphism theorem for rings. (8)

7. a) Prove that $M_i = M_1 \oplus M_2 \oplus + \dots \oplus M_n$ is an R module iff

i) $M = M_1 + M_2 + \dots + M_n$ and

ii) $M_1 \cap (M_1 + M_2 + \dots + M_{i-1} + M_{i+1} \dots M_n) = 0 \ \forall i,\ 1 \leq i \leq n$ (8)

b) Let K/F be a finite extension. Then show that K/F is an algebraic extension. (8)

8. a) Let $f(x) \in F[x]$ be of degree n. Then show that $f(x)$ has a splitting field (8)

b) Let F be a finite field with P^n elements. Then show that F has a subfield F^1 with P^m elements iff m divides n. (8)

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PGIS - N 1022 B - 14

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Ordinary differential Equations)

Paper - HCT-1.3

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates: Answer any **five** questions

All questions Carry equal Marks

1. a) For any real x_0 and constants α, β Show that there exists a solution of the initial value problem $y^{11} + a_1 y^1 + a_2 y = 0$ with $y(x_0) = \alpha, y^1(x_0) = \beta \; on - \infty < x < \infty$ **8**

   b) Prove that two solutions ϕ_1, ϕ_2 of L(y) = 0 are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I **8**

2. a) If $\alpha_1, \alpha_2 .....\alpha_n$ are any n constants and x_0 be any real number then prove that there exists a solution $\phi \; of \; L(y) = 0 \; on - \infty < x < \infty$ Satisfying $\phi(x_0) = \alpha_1, \phi^1(x_0) = \alpha_2, ...$ $\phi^{(n-1)}(x_0) = \alpha_n$ **8**

   b) If ϕ_1, ϕ_2 are two solutions of $y^{11} + a_1 y^1 + a_2 y = 0$ in an interval I containing a point x_0 then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} . W(\phi_1, \phi_2)(x_0)$ **8**

3. a) Define Green's function. Explain the method of finding green's function **8**

   b) Find the Green's function corresponding to differential Operator $L = \frac{d^2}{dx^2}$ with boundary conditions. $y(0) = o, y(1) = 0.$ **8**

4. a) State and prove sturm's separation theorem **8**

   b) Show that the boundary value problem $\frac{d^2 y}{dx^2} + \lambda y = 0 \; Where \; x(0) = 0, x(\pi) = 0$ is a sturm Liouville problem **8**

5. a) Define characteristic values and characteristic functions and find characteristic values and characteristic functions of $\frac{d}{dx}\left[x.\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0$

With $y^{1}(1)=0, y^{1}(e^{2\pi})=0, \lambda \geq 0$ **8**

b) Define adjoint and self adjoint equation. Further transform the following equation in to an equivalent self - adjoint equation $x^{2}y^{11}-2xy^{1}+2y=0$ **8**

6. a) Derive Picard's method of Successive approximations for the initial value problem

$y^{1}=f(x,y), y(x_{0})=y_{0}$ **8**

b) Find the third approximation using picards method of the equation $\frac{dy}{dx}=Z, \frac{dz}{dx}=x^{3}(y+z)$ where $y=1, Z=\frac{1}{2}$ at $x=0$ **8**

7. a) Write short note on Lipschitz, condition **4**

b) State and prove picard's existence theorem for the initial value problem

$\frac{dy}{dx}=f(x,y), y(x_{0})=y_{0}$ **12**

8. a) Explain the method of solving Riccati's equation when its one particular integral is known **8**

b) Solve the equation $x^{2}y_{1}+2-2xy+x^{2}y^{2}=0$ **8**

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PGIS - N 1028 B - 14
M.Sc. Ist Semester (CBCS) Degree Examination
Mathematics
(Fuzzy Sets And Fuzzy Systems)
Paper - SCT 1.2
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. *Answer any **Five** questions*
2. *All questions carry **equal** marks*

1. a) Define a fuzzy set and convexity of fuzzy set. Prove that a fuzzy set A on $\mathbb{R}$ is convex if and only if $A(\lambda x_1 + (1-\lambda)x_2) \geq \min\left[A(x_1), A(x_2)\right]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0,1]$ where min denote the minimum operator. **(8)**

b) Define the following and give an example of each

i) Strong $\alpha - Cut$

ii) Support set

iii) Level Set

iv) Convex fuzzy set **(8)**

2. a) Define a power fuzzy set. If $A, B \in f(x)$, then prove that, the following properties hold for all $\alpha \in [0,1]$ **(8)**

i) $^{\alpha}(A \cap B) = {^{\alpha}A} \cap {^{\alpha}B}$

ii) $^{\alpha+}(A \cup B) = {^{\alpha+}A} \cup {^{\alpha+}B}$

b) State and prove the first decomposition theorem. **(8)**

3. a) If $A_i \in F(X)$ for all $i \in I$, Where I is an index set, then prove that

i) $\bigcup_{i \in I} {}^{\alpha+} A_i = {}^{\alpha+}\left(\bigcup_{i \in I} A_i\right)$

ii) $\bigcap_{i \in I} {}^{\alpha+} A_i \leq {}^{\alpha+}\left(\bigcap_{i \in I} A_i\right)$ **(8)**

b) If $f: X \to Y$ be an arbitrary crisp function. Then for any $A \in F(X)$ and $B \in F(Y)$ prove that the following properties obtained by the extension principle hold

i) $f^{-1}(1-B) = 1 - f^{-1}(B)$

ii) $f(1-A) \geq 1 - f(A)$ **(8)**

4. a) Give the axiomatic definition of fuzzy compliment. If a function $C:[0,1] \to [0,1]$ Satisfy the axioms C_2 and C_4 then prove that C also Satisfies C_1 and C_3 and C is a bijection **(10)**

b) If C is a Continuous fuzzy compliment then prove that C has a unique equilibrium **(6)**

5. a) prove that the standard fuzzy intersection is the only idempotent t - norm **(8)**

b) Prove that for all $a, b \in [0,1]$ $i_{\min}(a,b) \leq i(a,b) \leq \min(a,b)$ Where $i_{\min}$ denotes the drastic intersection **(8)**

6. a) If i_w denote the class of yager t-norms defined by

$$i_w(a,b) = 1 - \min\left[1, \left[(1-a)^w + (1-b)^w\right]^{1/w}\right], w > 0$$

then prove that $i_{\min}(a,b) \leq i_w(a\ b) \leq \min\ (a,b)$ for all $a, b \in [0,1]$ **(8)**

b) If $< i, u, c >$ be a dual triple that satisfies the law of excluded middle and the law of contradiction, then prove that $< i, u, c >$ does not Satify the disributive laws **(8)**

7. a) Define a fuzzy number and explain the arithmetic operations on two fuzzy numbers A and B with suitable example. **(8)**

b) Write a note on lattice of fuzzy numbers. **(8)**

8. a) Define a fuzzy relation and explain with example. (4)

b) Write a note on projections and cylindrical extensions (4)

c) Define standard composition of two binary fuzzy relations P(X,Y) and Q(Y,Z)

If $P = [P_{ik}] = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}$

$$Q = [q_{kj}] = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$$

Then obtain the standard composition $R\,[X,Z]$ of P and Q (8)

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PGIS - N 1024 B - 14

M.Sc. Ist Semester(CBCS) Degree Examination

Mathematics

(Discrete Mathematics)

Paper : HCT - 1.4

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

i) Answer any **five** questions.

ii) All questions carry **equal** marks.

1. a) Define a Poset and a lattice. For any a and b in a lattice $(L,\leq)$ show that

i) $a \vee b = b$ *iff* $a \leq b$

ii) $a \wedge b = a$ *iff* $a \leq b$

iii) $a \wedge b = a$ *iff* $a \vee b = b$ **(5)**

b) If the join operation is distributive over the meet operation in a lattice, then prove that the meet operation is also distributive over the join operation. **(5)**

c) Define a boolean algebra. If $(L, \leq)$ is a boolean algebra, then for all a and b in L, Prove that

i) $(a \vee b)' = a' \wedge b'$

ii) $(a \wedge b)' = a' \vee b'$ **(6)**

2. a) Define a OR-gate, AND - gate and a NOT - gate giving the logic tables. **(5)**

b) Define

i) Min term ii) Max term.

Write the disjunctive normal form of the following functions. **(6)**

(x_1, x_2, x_3)	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
F_1	1	1	0	0	1	0	1	1
F_2	0	0	0	1	0	0	1	1

c) Discuss briefly about Relay in a switching circuit. (5)

3. a) Suppose that a valid computer password consists of a seven characters, the first of which is a letter chosen from the set {A,B,C,D,E,F,G} and the remaining six characters from the English alphabet or digit. How many different passwords are possible. (5)

b) For any two finite sets S and T Prove that $|S \cup T| = |S| + |T| - |S \cap T|$ (6)

c) Using the principle of inclusion and exclusiòn determine the number of integers between 1 and 250 that are divisible by any of the integers 2,3,5 and 7. (5)

4. a) Solve the Recurrence relation $a_r = a_{r-1} + a_{r-2}$ *with* $a_{0=1}$ *and* $a_{1=1}$ (5)

b) Find the particular Solution of $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ (5)

c) Solve $a_r = 3a_{r-1} + 2;\ r \geq 1$ with $a_0 = 1$ by the method of generating functions. (6)

5. a) Define

i) Weighted graph

ii) Multigraph.

Give examples for each (4)

b) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. (6)

c) If G is a directed graph, then prove that $\sum_{i=1}^{n} \deg_G + (v_i) = \sum_{i=1}^{n} \deg_G^- (v_i) = |E|$ (6)

6. a) Prove that there is one and only one path between every pair of vertices in a tree T.(6)

b) Prove that a tree with n vertices has (n-1) edges. (6)

c) Define transport network with an example. (4)

7. a) IF (S, *) and (T, *[1]) are monoids with identities e and e[1] respectively. if f: $S \to T$ is an isomorphism, then prove that f(e) = e[1]. (6)

b) If G is a group and a,b are elements of G, Then Prove that

i) $(a^{-1})^{-1} = a$

ii) $(ab)^{-1} = b^{-1}a^{-1}$ (4)

c) State and prove Lagrange's theorem on groups. (6)

8\. a) Write a note on Coding of binary in formation. (4)

b) Find the minimum distance of the (3,8) encoding function (6)

$e : B^3 \rightarrow B^8$ *defined by*

e (000) = 00000000
e (001) = 10111000
e (010) = 00101101
e (011) = 10010101
e (100) = 10100100
e (101) = 10001001
e (110) = 00011100
e (111) = 00110001

c) What is a group code. If $e : B^m \rightarrow B^n$ is a group code then show that the minimum distance of e is the minimum weight of a nonzero code word. (6)

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PGIS-N 1029 B-14
M.Sc. Ist Semester (CBCS) Degree Examination
Mathematics
(General Topology)
Paper - HCT-1.5
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

i) Answer any **five** questions.

ii) All questions carry **equal** marks.

1. a) Define closure of a set. Let A and B be subsets of a space X then prove the followings **(10)**

i) $\overline{A}$ is the smallest closed set containing A

ii) A is closed iff A=$\overline{A}$

iii) If A<B then $\overline{A} \subset \overline{B}$

iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

v) $(\overline{\overline{A}}) = \overline{A}$

b) Prove that $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$ and give an example to show that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$ **(6)**

2. a) Define limit point of a set. If A & B are subsets of a space X then prove the followings **(8)**

i) If $A \subset B$ *then* $D(A) \subset D(B)$

ii) $D(A \cup B) = D(A) \cup D(B)$

b) Define a subspace of a topological space. If (x, τ) be a topological space then show that a subspace of a subspace of a space is a subspace of the space **(8)**

3. a) Define a continuous mapping in a topological space. If f be a mapping of a space X into space Y. Let S be the subbase for the topology on Y then prove the followings are equivalent

i) If f is continuous (8)

ii) The inverse image of each member of S is open in X

b) Let X,Y be the topological spaces and $f: X \to Y$ be a mapping then show that f is closed iff $\overline{f(A)} \subset f(\overline{A})$ $for\ A \subset X$ (8)

4. a) Prove that a space (X,Z) is a T_0-space iff for each pair of distinct points x,y$\in$X, $\{\overline{X}\} \neq \{\overline{Y}\}$ (8)

b) Define a T_2-space. If X is T_2 and $f: X \to Y$ is a closed bijection then show that Y is also T_2-space (8)

5. a) Define a regular space. Show that regularity is a topological property (8)

b) Show that normality is a topological property (8)

6. a) State the axioms of countability. Let X be a first countable space and $A \subset X$ then prove that for a$\in$X is a limit point of A if and only if there exists a sequence $f: N \to A - \{a\}$ converging to 'a' (8)

b) Prove the following conditions are equivalent

i) The space X is connected

ii) The only subsets of X which are both open and closed are ϕ & X

iii) No continuous function $f: X \to \{0,1\}$ (8)

7. a) Define a compact space. Show that any continuous image of a compact space is compact (8)

b) State and prove the Heine-Boral theorem (8)

8. a) Prove that in any metric space the set of all open spheres is a base for topology on K (8)

b) Prove that a metric space is lindelof if and only if it is second countable (8)