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PGIVS-051 B-21

M.A./M.Sc. IV Semester (CBCS) Degree Examination

STATISTICS

Sampling Theory

Paper : HCT - 4.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **six** questions from Part - A and any **five** questions from Part - B.

Part - A (6×5=30)

1. What is PPS sampling? Give any two examples of it.
2. Explain Lahiri's method of drawing a sample with replacement.
3. Explain briefly Midzuno - sen sampling scheme.
4. Define two phase sampling. When is it useful?
5. Give the value of the intraclass correlation Coefficient, how do you choose between CSRSWOR and SRSWOR?
6. Define ratio and regression estimators, when are they used?
7. Distinguish between direct and synthetic estimators in small area estimation.
8. Discuss briefly non - sampling errors.

Part - B (5×10=50)

9. Define Des Raj estimator of the population total under PPSWOR scheme. Also find an unbiased estimator of the variance of this estimator.
10. Let $Y_1, Y_2, \ldots\ldots, Y_n$ be an PPSWOR samples and let $P_1, P_2, \ldots\ldots\ldots P_n$ be the corresponding associates probabilities prove that

    a. Eti = y, the population total $\forall i = 1, 2, \ldots\ldots\ldots, n$.

    b. $t_1, t_2, \ldots\ldots\ldots t_n$ are uncorrelated where

    $$t_1 = \frac{Y_1}{P_1} \text{ and}$$

    $$t_1 = Y_1 + Y_2 + \ldots\ldots + Y_{i-1} + \frac{Yi}{Pi}(1 - P_1 - P_2, \ldots\ldots P_{i-1}) \quad i = 2,3,\ldots\ldots\ldots,n.$$

11. Outline cluster sampling with equal sizes. Derive an unbiased estimator of population mean under this scheme and obtain its variance. In terms of intraclass correlation coefficient.

12. What is two stage sampling? For SRSWOR as sample design at both stages, obtain an unbiased estimator of the population total and its variance.

13. a. In two phase sampling, obtain an unbiased estimator of the population total when SRSWOR is used at the first phase and PPSWR is used at the second phase.

    b. Discuss optimum allocation in two stage sampling when the budget of the survey is fixed.

14. Giving examples for sensitive issues, explain the randomized response technique. Discusses the Warner's model for estimating a proportion.

15. Clearly stating the assumptions, obtain expressions for the bias and mean square error of ratio estimator the population ratio.

16. Write short notes on any **two** of the following.

    a. Cumulative total method.

    b. Repetitive surveys.

    c. Modelling observational errors.

    d. Almost unbiased ratio estimators.

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PGIVS-054-B-21

M.A./M.Sc. IV Semester (CBCS) Degree Examination

STATISTICS

Practical based on HCT 4.1

Paper - HCP - 4.1

Time : 2 Hours **Maximum Marks : 30**

Instructions to Candidates:

Answer any TWO questions.

1. Obtain unbiased estimator of the gain in efficiency of PPSWR over SRSWR in estimating the population of UP using the following data by taking

   i) A SRSWR sample of size 5 and

   ii) A PPSWR sample of size 5 using cumulative total method.

District No.	1	2	3	4	5	6	7	8
Population in lakh (y)	17.28	10.86	16.16	18.6	23.72	14.98	17.36	4.3
Area in miles (x)	1543	3888	2236	1227	2084	2620	1887	1201

2. A pilot sample survey for the cultivation practices and yield of Guava was conducted out of 412 bearing trees 5-cluster of 4-trees each were selected using SRSWOR and yield in Kg where recorded as follows.

Clusters	Trees			
	1	2	3	4
01	5.53	4.84	0.69	15.79
02	11.08	10.93	19.08	11.18
03	12.06	0.65	4.21	7.56
04	0.87	32.56	16.92	37.02
05	6.4	3.56	4.81	57.54

Estimate the gain in efficiency of CSRSWOR over SRSWOR. Also estimate ρ_c.

3. In a village there are 8 orchards with respective yields 60, 35, 30, 44, 30, 50, 22 and 40. The corresponding number of trees in these orchards were 50, 30, 25, 40, 26, 44, 20 and 35.

   i. Select a random sample of size 2 using PPSWOR by Lahiri's method.

   ii. Estimate along with standard error the total production of 8 orchards by Hurwitz - Thompson and Des - raj methods.

4. For studying milk yield, feeding and management practices of milk animals in the year 1977-78 of Haryana state, 5 villages were selected with SRSWR. The total numbers of milk animals in 1977-78 along with their livestock census data in 1976 are given below.

Sl. No. of the village	01	02	03	04	05
No. of Milk animals in survey (Y)	1129	1144	1125	1138	1137
No. of Milk animals in Census (X)	1141	1144	1127	1153	1117

Estimate the total number of milk animals in 117 villages of Haryana by

i. Ratio method

ii. Regression method.

Given that the total number of milk animals in the census was 143968. Compare the two estimators by computing their standard errors.

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PGIVS-053 B-21

M.A./M.Sc. IV Semester (CBCS) Degree Examination

STATISTICS

(Operations Research in Industries)

Paper - SCT - 4.1(a)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any six questions from Part - A and any five questions from Part - B.

Part - A (6×5=30)

1. Discuss briefly about the models in Operations research.
2. Define
   a. Feasible solution.
   b. Infeasible solution.
   c. Unbounded solution.
3. Explain Big - M method of solving LPP.
4. Explain the Wolfe's method in QPP.
5. Define integer programming problem. Distinguish between pure and mixed integer programming problems.
6. Define,
   a. Node.
   b. Activity.
   c. Network.
7. Discuss (t_p, s) System of inventory problem.
8. Define purchase price, Scheduling period and Lead time of inventory problem.

Part - B (5×10=50)

9. Define Operations Research (OR) what is its scope? Explain briefly.

10. Solve the following LPP by using Simplex method.

Max $Z = x_1 + 3x_2 - 2x_3$

S.to $3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1 x_2, x_3 \geq 0$

11. Solve the following NLPP.

Optimize $z = x_1^2 + x_2^2 + x_3^2$

S.to $4x_1 + x_2 + 2x_3 = 14$

$x_1 x_2, x_3 \geq 0$

12. Explain Beal's method for solving Quadratic Programming Problem.

13. Outline PERT technique of solving network problem.

14. A project has the following characteristics

Activity :	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-7	7-8	8-10	9-10
Time (Days) :	4	1	1	1	6	5	4	8	1	2	5	7

Construct the network, obtain the critical path and minimum duration of the project.

15. Discuss Harry's model of an inventory problem.

16. Write short notes on any two of the following.

a. Simplex method of solving LPP.

b. Dual of the dual is primal.

c. Quadratic Programming problem.

d. Inventory models with probabilistic demand.

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PGIVS-056 B-21

M.A./M.Sc. IV Semester (CBCS) Degree Examination

STATISTICS

Practical Based on SCT : 4.1 (a)

Paper - SCP - 4.1 (a)

Time : 2 Hours **Maximum Marks : 30**

Instructions to Candidates:

i. Answer any **two** questions.

ii. Each question carries **15** marks.

1. Solve the following LPP by Simplex method.

Max $Z = 12x_1 + 8x_2$

Subject to $5x_1 + 2x_2 \leq 150$

$2x_1 + 3x_2 \leq 100$

$4x_1 + 2x_2 \leq 80$

and $x_1, x_2 \geq 0$

2. Solve the following LPP using two - phase method.

Max $Z = 12x_1 + 18x_2 + 15x_3$

Subject to $4x_1 + 8x_2 + 6x_3 \geq 64$

$3x_1 + 6x_2 + 12x_3 \geq 96$

and $x_1, x_2, x_3 \geq 0$.

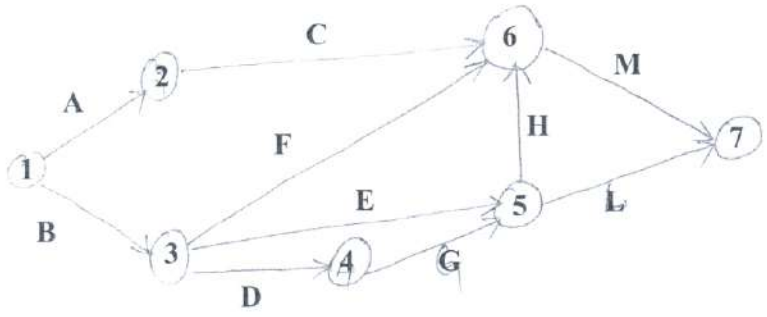
3. Solve the following NLPP using Lagrange's method.

Max $Z = x_1^2 + 2x_2^2 + 3x_3^2$

Subject to $2x_1 + x_2 + 2x_3 = 30$

and $x_1, x_2, x_3 \geq 0$.

4. Consider the following PERT system flow network.



Three Estimates for the time for each activity are as follows.

Activity :	A	B	C	D	E	F	G	H	L	M
Latest time (t_0)	8	6	9	5	8	11	3	5	8	4
Greatest time (t_l)	10	9	15	5	11	20	6	8	12	10
Most likely time (t_m)	9	7	12	5	10	15	4	6	10	5

Determine the following :

i. Expected time for each activity and variance.

ii. The earliest and latest expected time to accomplish each event.

iii. Critical path.

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PGIVS-052 B-21

M.A./M.Sc. IV Semester Degree Examination

STATISTICS

(Multivariate Analysis)

Paper : HCT - 4.2

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **six** questions from Part - A and any **five** questions from Part - B.

Part - A (6×5=30)

1. Obtain the distribution of an orthogonal transformation CX where X has $N_p(\mu,\Sigma)$ distribution.
2. If a random vector $X'=(X^{(1)},X^{(2)})$ has $N_p(\mu,\Sigma)$ and $X^{(1)}$ is a q-components vector, then show that $X^{(1)}$ and $X^{(2)}$ are independent if $\Sigma_{12}=0$.
3. Obtain the distribution of sample mean vector when the sample is taken from multivariate normal distribution.
4. Define Hotelling's T^2 and Mahalanobis' D^2 statistic. Establish the relation between them.
5. Obtain the characteristics function of P - variate normal random vector.
6. Discuss the problem of classification. Give one example.
7. Let X_α follows $N_p(\beta Z_\alpha,\Sigma)$ distribution when $Z_1, Z_2,\ldots\ldots, Z_n$ are known vectors. Find the MLE of β.
8. Define Canonical correlations. Show that multiple correlation coefficient is a particular case of canonical correlations.

Part - B (5×10=50)

9. Show that the conditional distribution obtained from a multivariate normal distribution is multivariate normal.
10. Define multivariate normal distribution. Obtain the maximum likelihood estimators of μ and Σ.

11. Derive Hotelling's T^2 statistic as a likelihood ratio statistic to test for the mean vector of $N_p(\mu, \Sigma)$, when Σ is unknown. State the null distribution of this statistic.

12. Derive likelihood ratio test for testing $H_0 : \Sigma = \Sigma_0$ on the basis of a random sample from $N_p(\mu, \Sigma)$ distribution.

13. Discuss the analysis of two - way classified data in the multivariate normal case.

14. Define the principal components of a random vector X whose var - covariance matrix is Σ. Expalin how you obtain them.

15. Show that the population canonical correlations are the roots of the determinantal equation $\left|\Sigma_{12}\Sigma_{22}\Sigma_{21} - \lambda^2 \Sigma_{11}\right| = 0$.

16. Derive the likelihood ratio test for testing $H_0 : \Sigma = \sigma^2 \Sigma_0$ where σ^2 is unknown and Σ_0 is a given matrix, using a random sample from $N_p(\mu, \Sigma)$.
