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PGIIIS-O- 834 B-19

M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Operations Research-II

Paper : OET 3.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) Use Big-M method to solve the following L.P.P.

Minimize $z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

and $x_1, x_2 \geq 0$ **(8)**

b) Use two -phase simplex method to solve the following L.P.P.

Maximize $z = x_1 + 2x_2 + 3x_3$

Subject to $x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_1 - x_3 + \geq 2$

and $x_1, x_2, x_3 \geq 0$ **(8)**

2. a) Prove that dual of the dual is primal. **(8)**

b) Solve the following L.P.P by dual simplex method

Minimize $z = 5x_1 + 6x_2$

Subject to $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

And $x_1, x_2 \geq 0$ **(8)**

3. a) Explain

i. Two- Person zero -sum games.

ii. The Maximum- Minimax principle. **(8)**

b) Solve the following game whose payoff matrix is given by

Player B

Player A $\begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix}$ **(8)**

4. a) Solve the following game graphically

Player B

Player A $\begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix}$ **(8)**

b) Using the principle of abminance, solve the following game

Player B

Player A $\begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 \end{matrix} \\ \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \end{bmatrix} \end{array}$ **(8)**

5. a) Explain a queueing system with its characteristics. **(8)**

b) Explain departure process. **(8)**

6. a) Discuss $(M/M/1):(\infty/FIFO)$ model. **(10)**

b) In a Railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter- arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate

i. The mean queue size (in length)

ii. The probability that the queue size exceeds 10.

iii. If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii). **(6)**

7. a) Discuss event-type Simulation. **(8)**

b) Discuss Simulation languages. **(8)**

8. a) Discuss the Monte-carlo Simulation technique. **(8)**

b) What are the advantages and limitations of Simulation models? **(8)**

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PGIIIS-N-833 B-19

M.Sc. III Semester (CBCS) Degree Examination

MATHEMATICS

Fluid Mechanics-I

Paper : SCT-3.1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

i) Answer any five questions.

ii) All questions carry equal marks.

I. a) Derive boundary conditions on the velocity, pressure and temperature. **(8)**

b) Derive symmetrical form of equations of continuity. **(8)**

2. a) Prove that liquid motion is possible when velocity at (x, y, z) is given by $u = \frac{3xz}{r^5}, v = \frac{3yz}{r^5}, w = \frac{3z^2 - r^2}{r^5}$ where $r^2 = x^2 + y^2 + z^2$ and the stream lines are the intersection of the surfaces $\left(x^2 + y^2 + z^2\right)^3 = c\left(y^2 + z^2\right)^2$ by the planes passing through ox and also the velocity potential is given by $\frac{\cos\theta}{r^2}$. **(8)**

b) Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2}\phi(t) - 1 = 0$ where $f(t).\phi(t)$ =constant, is possible form of the boundary surface of a liquid. **(8)**

3. a) Derive Lagranges equation of motion. **(8)**

b) State and prove Kelvins circulation theorem. **(8)**

4. a) Find the complex potential for a two-dimensional source of strength m placed at the origin. **(8)**

b) Find the lines of flow in two-dimensional fluid motion given by

$$\phi + i\,\psi = -\frac{n}{2}(x+iy)^2 e^{2int}$$

verify that the paths of the particles of the fluid may be obtained by eliminating t from the equations.

$$r\cos(nt+\theta) - x_0 = r\sin(nt+\theta) - y_0 = nt(x_0 - y_0)$$ **(8)**

5. a) State and prove Blasius theorem. **(8)**

b) Derive the equations for the general motion of any cylinder. **(8)**

(OR)

6. a) A circular cylinder is placed in a uniform stream, find the forces acting on the cylinder. **(8)**

b) A circular cylinder of radius a and infinite length lies on a plane in an infinite depth of liquid. The velocity of liquid at a great distance from the cylinder is U, perpendicular to generators and the motion is irrotational and two dimensional. Verify that the stream function is the imaginary part of $W = \pi\alpha U \coth\left(\frac{\pi a}{z}\right)$, *where* $z = 0$ on the line of contact. Prove that the pressure at the two ends of the diameter of the cylinder normal to the plane differs by $\frac{1}{32}\pi^4 \rho U^2$ **(8)**

7. a) Prove that the necessary and sufficient condition for irrotational motion is that there exists a potential ϕ such that $\vec{q} = -\nabla\phi$, q being velocity vector. **(8)**

b) State and prove Kelvins minimum energy theorem. **(8)**

(OR)

8. a) Show that in the motion of fluid in two-dimensions, if the coordinates (x,y) of an element at any time can be expressed in terms of initial coordinates (a,b) and the time. The motion is irrotational if $\frac{\partial(\dot{x},x)}{\partial(a,b)} + \frac{\partial(\dot{y},y)}{\partial(a,b)} = 0$ **(8)**

b) Show that under certain conditions the motion of a frictionless fluid if once irrotational will always be so, is true also when each particle is acted upon by a resistance varying as the velocity. **(8)**

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PGIIIS-O 831 B-19

M.Sc. III Semester Degree Examination

MATHEMATICS

Graph Theory - I

Paper - HCT 3.2

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** full questions.
2. All questions carry **equal** marks.

1. a) Prove that the number of vertices of odd degree in a graph is always even. **(5)**

b) Define cut vertex and bridge in a connected graph. Draw a graph with more cut vertices than bridges. **(5)**

c) Prove that an edge 'e' of a connected graph G is a bridge of G if and only if 'e' does not lie on a cycle of G. **(6)**

2. a) Show that a graph G is bipartite if and only if all its cycles are even. **(6)**

b) Construct cubic graphs with 2n vertices, where $3 \le n \le 6$ having no triangle. **(4)**

c) Show that for any graph G with six vertices G or $\overline{G}$ contains a cycle. **(6)**

3. a) Prove that a connected graph G is a tree if and only if every edge is a bridge. **(6)**

b) Define a spanning tree in a connected graph. Show that a graph G is connected if and only if it contains a spanning tree. **(6)**

c) Show that every non trivial tree contains atleast two end vetices. **(4)**

4. a) Explain the meaning of eccentricity, radius and diameter of a graph through a graph. Show that for every connected graph G, $rad(G) \le dia(G) \le 2rad(G)$. **(8)**

b) Define Rank and Nullity in a spanning tree. Prove that a graph is a tree if and only if it is minimally connected. **(8)**

5. a) Define vertex and edge connectivity of a graph with an example. Prove that in any graph $k(G) \le \lambda(G) \le \delta(G)$. **(10)**

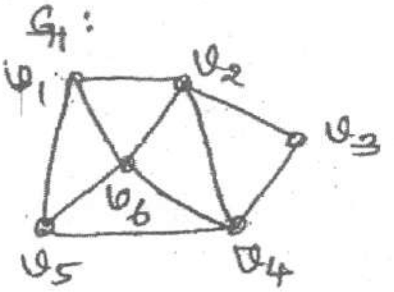
b) State the Graphical variations of Menger's theorem and explain through a graph. **(6)**

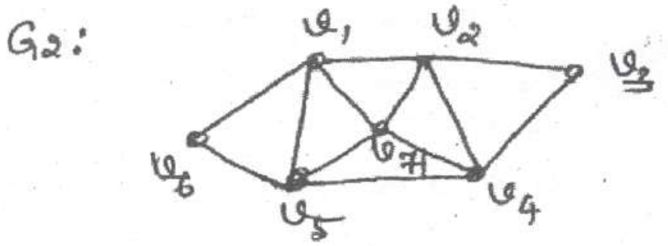
6. a) For every non trivial connected eulerian graph G, show that the set of edges of G can be partitioned into cycles. Also find for what values of 'n' does k_n, a complete graph with n - vertices have an euler circuit. **(10)**

b) Find under what conditions the complete bipartite graph $k_{m,n}$ has an eulerian circuit. **(6)**

7. a) If G is a graph with $p \ge 3$ vertices such that for all non adjacent vertices u and v, $\deg u + \deg v \ge p$, then show that G is Hamiltonian. **(8)**

b) Determine which of the following graphs have eulerian trail and eulerian circuit. Explain.





4)

c) Draw the following graphs :

i. An Eulerian graph which is not Hamiltonian.

ii. An Eulerian graph which is also Hamiltonian. **(4)**

8. a) Define line graph of a graph. Show that a graph G is the line graph of a tree if and only if it is a connected block graph in which each cutvertex is on exactly two blocks. **(8)**

b) Define a tournament. Prove that every tournament has a Hamiltonian path. **(8)**

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PGIIIS-N-831 B-19
M.Sc. III Semester Degree Examination
MATHEMATICS
Graph Theory - I
Paper : HCT 3.2
(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **five** full questions.
2. All questions carry **equal** marks.

1. a) Define a regular graph. Construct an r - regular graph with 8 vertices for each $0 \le r \le 5$. **(7)**

b) Prove that for any graph G with six vertices G or $\overline{G}$ contains a cycle. **(4)**

c) Define join and product operations. Draw join and product of the following graphs :

i. $\overline{C}_4 + \overline{k}_3$

ii. $k_{1,2} + \overline{P}_2$

iii. $k_3 \times k_3$

iv. $k_{1,3} \times k_{1,3}$ **(5)**

2. a) Prove that a graph is planar if and only if it does not have any subdivision of k_5 or $k_{3,3}$. **(8)**

b) Define a maximal planar graph. If G is a maximal planar (p,q) graph with $p \ge 3$ then show that q = 3p-6. **(8)**

3. a) Show that the petersen graph is nonplanar. **(4)**

b) Define self dual graph. For any connected graph G prove that $G \cong G^{**}$. **(6)**

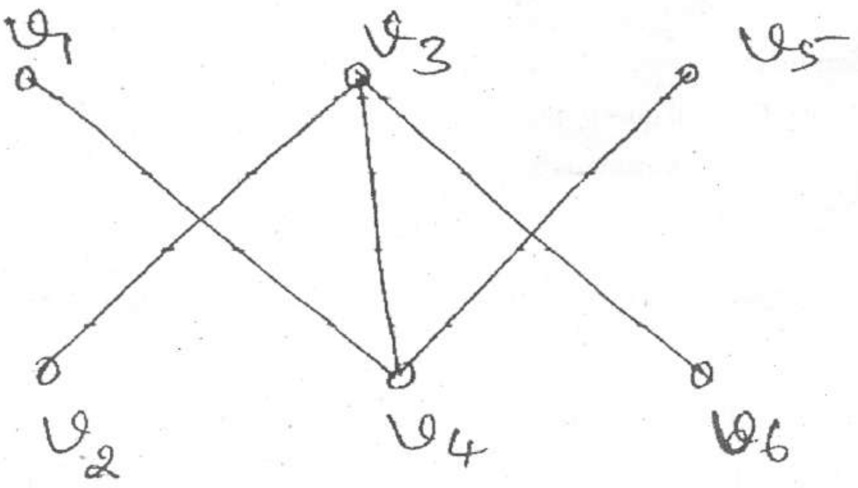
c) Define crossing number and thickness of a graph. Determine $Cr(k_{2,2,2})$. **(6)**

4. a) For any graph G, show that $X(G) \leq 1 + \max \delta(G')$, where the maximum is taken over all induced subgraphs G' of G. **(6)**

b) Find the chromatic number of cubic graphs with $P = 8, 10$ vertices with triangles and without triangles. **(4)**

c) For any graph G, prove the inequality, $2\sqrt{p} \leq \chi + \overline{X} \leq p+1$. **(6)**

5. a) Write the steps for Welsh and Powell algorithm. Also illustrate this algorithm to the following graph. **(10)**



b) Write the steps for the smallest last sequential algorithm. (6)

6. a) Prove that a map is k - face colorable if and only if its dual G* is k - vertex colorable. **(8)**

b) Show that a map is 2 - face colorable if and only if it is an Eulerian graph. **(8)**

7. a) Define a factor of a graph. Draw a 2 - factorization of k_5. Show that a complete graph k_{2n} is 1 - factorable. **(8)**

b) Show that for every positive integer n, the graph k_{2n+1} can be factored into n-Hamiltonian cycles. **(8)**

8. a) Define symmetric and Asymmetric digraph. Prove that a digraph G is an Euler digraph if and only if G is connected and is balanced. **(8)**

b) Define a tournament. Prove that every tournament has a Hamiltonian path. **(8)**
