

Roll No. \_\_\_\_\_

[Total No. of Pages : 4

**PGIS-N-235 A-21**  
**M.A./M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Operations Research**  
**Paper : SCT - 1.1**  
**(New)**

**Time : 3 Hours**

**Maximum Marks : 80**

*Instructions to Candidates:*

- i. Answer any **Five** questions.
- ii. All questions carry **equal** marks.

1. a) Explain briefly the general methods for solving O.R. models. (8)
- b) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirting and woolens, yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per meter respectively. one meter suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. One meter shirting requires 4 minutes in weaving, 1 minute in Processing and 3 minutes in packing, while one meter woolen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours of weaving, processing and packing departments respectively. Formulate the problem as a linear programming problem. (8)
2. a) Solve the following L.P.P. by graphical method (8)

$$\text{Maximize } Z = 2x_1 + 3x_2$$

$$\text{Subject to } x_1 + x_2 \leq 30$$

$$x_2 \geq 3$$

$$0 \leq x_2 \leq 12$$

$$0 \leq x_1 \leq 20$$

$$\text{and } x_1, x_2 \geq 0$$

- b) Solve the following L.P.P. by simplex method (8)

$$\text{Maximize } Z = 4x_1 + 10x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$\text{and } x_1, x_2 \geq 0$$

3. a) Use Big - M method to solve the following L.P.P. (8)

$$\text{Minimize } Z = 5x_1 + 3x_2$$

$$\text{Subject to } 2x_1 + 4x_2 \leq 12$$

$$2x_1 + 2x_2 = 10$$

$$5x_1 + 2x_2 \geq 10$$

$$\text{and } x_1, x_2 \geq 0$$

- b) Use two - phase simplex method to solve the following L.P.P. (8)

$$\text{Minimize } Z = x_1 + x_2$$

$$\text{Subject to } 2x_1 + x_2 \geq 4$$

$$x_1 + 7x_2 \geq 7$$

$$\text{and } x_1, x_2 \geq 0$$

4. a) Write the dual of the following L.P.P. (6)

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{Subject to } 3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$\text{and } x_1, x_2 \geq 0$$

- b) Use dual simplex method to solve the following L.P.P. (10)

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{Subject to } x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 10$$

$$x_2 \leq 3$$

$$\text{and } x_1, x_2 \geq 0$$

5. a) Explain the method of column minima and row minima to obtain initial basic feasible solution to the given transportation problem. (10)
- b) Determine the initial basic feasible solution to the following transportation problem by using North - west corner rule (6)

**(Destinations)**

		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
<b>(Origins)</b>	O <sub>1</sub>	6	4	1	5	14
	O <sub>2</sub>	8	9	2	7	16
	O <sub>3</sub>	4	3	6	2	5
		6	10	15	4	

**(Availability)**

**(Requirements)**

6. a) Write the transportation technique. (4)
- b) Find the optimum transportation schedule for the following problem by using transportation (12)

**(Destinations)**

		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
<b>(Source)</b>	O <sub>1</sub>	19	30	50	10	7
	O <sub>2</sub>	70	30	40	60	8
	O <sub>3</sub>	40	8	40	20	18
		5	8	7	14	

**(Supply)**

**(Demand)**

7. a) Solve the following assignment problem (8)

	A	B	C	D
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

- b) Given the following matrix of set up costs, show how to sequence production so as to minimize set up cost per cycle. (8)

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

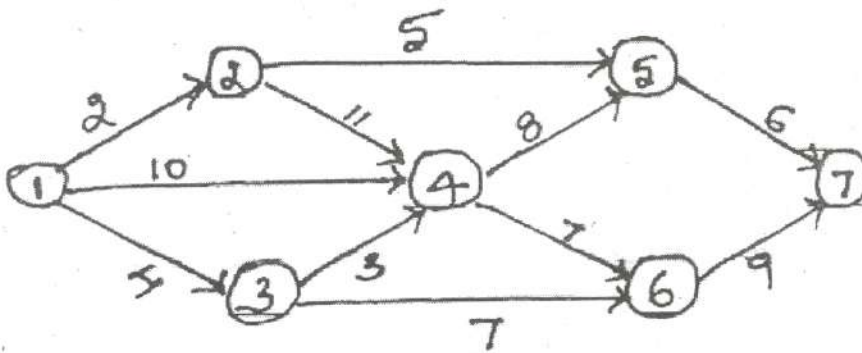
8. a) Solve the following game graphically

(8)

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

b) For the following network, find the shortest route from node 1 to node 7.

(8)



Roll No. \_\_\_\_\_

[Total No. of Pages : 2

**PGIS-O-235 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Classical Mechanics**  
**Paper : SCT - 1.1**  
**(Old)**

**Time : 3 Hours**

**Maximum Marks : 80**

**Instructions to Candidates:**

Answer any **Five** full questions.  
All questions carry **equal** marks.

1. a) State and prove D'Alembert's principle. (8)  
b) Derive Lagrange's equation for impulsive motion. (8)
2. a) Obtain Euler's dynamical equations of motion of a rigid body about a fixed point on the body. (8)  
b) Prove that  $M_z = \frac{\partial L}{\partial \dot{\phi}_i}$  for a system with  $L = \frac{1}{2} \sum_i m_i (\dot{r}_i^2 + r_i^2 \dot{\phi}_i^2 + \dot{z}_i^2) - V(r)$ . (8)
3. a) Obtain Hamilton's equation of motion of a particle in a plane referred to moving axes, where the components of velocities are  $u = \dot{x} - \omega y, v = \dot{y} + \omega x$ . (8)  
b) Deduce Lagrange's and Hamilton equations from Hamilton's principle. (8)
4. a) Derive Hamilton's canonical equation from Hamilton's principle. (8)  
b) Deduce Poincare integral invariant. (8)
5. a) Show that the transformation given by  $q = \sqrt{2P} \sin Q, p = \sqrt{2P} \cos Q$  is canonical by using Poisson's bracket. (8)  
b) Deduce Whittaker's equations. (8)
6. a) Prove that Poisson's bracket are also invariant under canonical transformation. (8)  
b) Show that Poisson's bracket of two constants of motion is itself a constant of motion. (8)

7. a) A particle of mass  $m$  falling under gravity, solve for the motion of the particle using canonical transformation theory. (8)
- b) Derive Hamilton - Jacobi equation. (8)
8. a) Show that the Lagrange's bracket do not obey the commutative law. Also prove the fundamental Lagrange's bracket. (8)
- b) Find the relationship between Lagrange's and Poisson's bracket. (8)
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**PGIS-N-230 A-21**  
**M.A./M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Real Analysis**  
**Paper : HCT - 1.1**  
**(New)**

Time : 3 Hours

Maximum Marks : 80

**Instructions to Candidates:** Answer any **Five** full questions.  
 All Questions carry **equal** marks.

1. a) If  $P^*$  is a refinement of  $P$ , then show that  $L(P^*, f, \alpha) \geq L(P, f, \alpha)$  and  $U(P^*, f, \alpha) \leq U(P, f, \alpha)$ . (8)
- b) If  $f \in \mathbb{R}(\alpha_1)$  and  $f \in \mathbb{R}(\alpha_2)$ . Then prove that  $f \in \mathbb{R}(\alpha_1 + \alpha_2)$  and  $\int_a^b f d(\alpha_1 + \alpha_2) = \int_a^b f d\alpha_1 + \int_a^b f d\alpha_2$ . (8)
2. a) Suppose  $f \in \mathbb{R}(\alpha)$  on  $[a, b]$ ,  $m \leq f \leq M$ ,  $\phi$  is continuous on  $[m, M]$  and  $h(x) = \phi[f(x)]$  on  $[a, b]$ . Then show that  $h \in \mathbb{R}(\alpha)$  on  $[a, b]$ . (8)
- b) If  $f$  and  $\phi$  are continuous on  $[a, b]$ ,  $\phi$  is strictly increasing on  $[a, b]$  and  $\chi$  is inverse function of  $\phi$ . Then prove that  $\int_a^b f(x) dx = \int_{\phi(a)}^{\phi(b)} f(\chi(y)) d\chi(y)$ . (8)
3. a) If  $f \in \mathbb{R}$  on  $[a, b]$  and  $\alpha$  is monotone increasing on  $[a, b]$  such that  $\alpha' \in \mathbb{R}$  on  $[a, b]$ , then show that  $f \in \mathbb{R}(\alpha)$  and  $\int_a^b f d\alpha = \int_a^b f \alpha' dx$ . (8)
- b) If a sequence  $\{f_n\}$  converges uniformly in  $[a, b]$  and  $x_0$  is a point of  $[a, b]$ . Such that  $\lim_{n \rightarrow \infty} f_n(x) = a_n$ ,  $n = 1, 2, \dots$  then prove that
  - i.  $\langle a_n \rangle$  converges
  - ii.  $\lim_{x \rightarrow x_0} f(x) = \lim_{n \rightarrow \infty} a_n$ . (8)

4. a) Let a series  $\sum f_n$  of differentiable functions converges point wise to  $f$  on  $[a,b]$  and each  $f'_n$  is continuous on  $[a,b]$  and the series  $\sum f'_n$  converges uniformly to  $G$  on  $[a,b]$  then prove that the given series  $\sum f_n$  converges uniformly to  $f$  on  $[a,b]$  and  $f'(x) = G(x)$ . (8)
- b) If  $k$  is compact if  $f_n \in G(k)$  for  $n = 1, 2, \dots$  and if  $\{f_n\}$  is point wise bounded and equicontinuous on  $k$ . then show that
- $\{f_n\}$  is uniformly bounded on  $k$
  - $\{f_n\}$  contains a uniformly convergent subsequence. (8)
5. State and prove Stone - Weierstrass theorem. (16)
6. a) If a power series  $\sum a_n x^n$  converges at the end point  $x = R$  of the interval of convergence  $(-R, R)$ , then prove that it is uniformly convergent in the closed interval  $[0, R]$ . (8)
- b) If a function  $\phi$  is bounded and integrable on the interval  $[a, b]$ , then show that as  $n \rightarrow \infty$
- $$A_n = \int_a^b \phi \cos nxdx \rightarrow 0 \text{ and}$$
- $$B_n = \int_a^b \phi \sin nxdx \rightarrow 0 \quad (8)$$
7. a) If  $f$  is bounded and integrable on  $[-\pi, \pi]$  and if  $a_n, b_n$  are its Fourier coefficients, then prove that  $\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$  converges. (8)
- b) Define gamma function. Find  $\Gamma(1/2) = \sqrt{\pi}$ . (8)
8. a) Prove that a linear operator  $T$  on a finite dimensional vector space  $X$  is one - to - one if and only if the range of  $T$  is all of  $X$ . (8)
- b) If  $f$  is differentiable at  $x_0$ , then show that  $f$  is continuous at  $x_0$ . (8)



Roll No. \_\_\_\_\_

[Total No. of Pages : 2

**PGIS-236 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Fuzzy Sets and Fuzzy Systems**  
**Paper : SCT - 1.2**  
**(Old)**

**Time : 3 Hours**

**Maximum Marks : 80**

**Instructions to Candidates:**

1. Answer any **Five** full questions.
2. All Questions carry **Equal** marks.

1. a) State and prove De-Morgan's laws for crisp sets. (6)  
b) Explain the notion of fuzzy set with suitable example. (5)  
c) Define  $\alpha$  - cut of a fuzzy set and explain with an example. (5)
2. a) Define a convexity of fuzzy set and prove that a fuzzy set  $A$  on  $R$  is convex if and only if  $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$  for all  $x_1, x_2 \in R$  and  $\lambda \in [0, 1]$ . (8)  
b) Define standard fuzzy set operations and explain with examples. (8)
3. a) If  $A, B$  are fuzzy sets defined on the universal set  $X$ , then prove that the following holds for all  $\alpha, b \in [0, 1]$ .
  - i.  ${}^\alpha(A \cap B) = {}^\alpha A \cap {}^\alpha B$
  - ii.  ${}^\alpha(A \cup B) = {}^\alpha A \cup {}^\alpha B$  (8)  
b) If  $A, B \in F[X]$ , then show the properties hold for all  $\alpha \in [0, 1]$ 
  - i.  $A \subseteq B$  iff  ${}^\alpha A \subseteq {}^\alpha B$
  - ii.  $A \subseteq B$  iff  ${}^{\alpha+} A \subseteq {}^{\alpha+} B$  (8)

4. a) State and prove second decomposition theorem. (8)
- b) If a function  $C : [0,1] \rightarrow [0,1]$  satisfy the axioms  $C_2$  and  $C_4$  of fuzzy complements then prove that it also satisfies axioms  $C_1$  and  $C_3$ . (8)
5. a) If  $C$  is a continuous fuzzy complement then show that  $C$  has a unique equilibrium. (8)
- b) Write the axiomatic skeleton for  $t$  - norms. Prove that for all  $a, b \in [0,1]$   
 $i_{\min}(a, b) \leq i(a, b) \leq \min(a, b)$  where  $i_{\min}$  denotes the drastic intersection. (8)
6. a) If  $i_w$  denote the class of Yager  $t$  - norms defined by  
 $i_w(a, b) = 1 - \min[1, (1-a)^w + (1-b)^w]$ ,  $w > 0$  then show that  
 $i_{\min}(a, b) \leq i_w(a, b) \leq \min(a, b)$  for all  $a, b \in [0,1]$ . (8)
- b) Write axiomatic definition of  $t$ -conorms and write any one parameterize class of increasing generator and the corresponding class of  $t$  - conorms. (8)
7. a) If  $(i, u, c)$  is a dual triple that satisfies the law of excluded middle and law of contradiction, then prove that  $\langle i, u, c \rangle$  does not satisfy the distributive law. (8)
- b) Define a fuzzy number and write an example and also discuss special cases of fuzzy numbers. (8)
8. a) Define Linguistic variables and explain with suitable example. (8)
- b) Write a detailed note on fuzzy equations. (8)

Roll No. \_\_\_\_\_

[Total No. of Pages : 2

**PGIS-N-231 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Algebra - I**  
**Paper : HCT - 1.2**  
**(New)**

**Time : 3 Hours**

**Maximum Marks : 80**

**Instructions to Candidates:**

1. Answer any **Five** questions.
2. All Questions carry **Equal** marks.

1. a) Let  $N$  be a normal subgroup of a group  $G$  and  $H$  a subgroup of  $G$ . Then prove that  $NH$  is a subgroup of  $G$ . (8)
- b) Prove that every quotient group of a cyclic group is cyclic but not conversely. (8)
2. a) If  $G = H_1 \otimes H_2$  i.e.,  $G$  is the internal direct product of its subgroups  $H_1$  and  $H_2$ , then prove that
  - i.  $H_1$  and  $H_2$  are normal subgroups of  $G$ .
  - ii.  $G/H_1 = H_2$ ,  $G/H_2 = H_1$ . (8)
- b) Suppose  $H$  and  $K$  are subgroups of a finite group  $G$ . Also let  $O(H) > \sqrt{O(G)}$ ,  $O(K) > \sqrt{O(G)}$  then show that  $H \cap K = \{e\}$ . (8)
3. a) If a  $G$  is an abelian group of order  $O(G)$ , and if  $P$  is a prime number such that  $P^\alpha \mid O(G)$ ,  $P^{\alpha+1} \nmid O(G)$ , then prove that  $G$  has a subgroup of order  $P$ . (8)
- b) State and prove third sylow theorem. (8)
4. a) Prove that a group  $G$  is solvable iff  $G^{(n)} = e$  for some  $n \geq 1$ . (8)
- b) State and prove Scheiers theorem. (8)

5. a) Define ring, Euclidean domain, Let  $R$  be is Euclidean domain. Then show that  $a \in R$  is a unit iff  $d(a) = d(1)$ . (8)
- b) State and prove unique factorisation domain. (8)
6. a) If  $P$  is a prime number of the form  $4n+1$ , then we can solve the congruence  $x^2 \equiv -1 \pmod{p}$ . (8)
- b) State and prove FERMAT theorem. (8)
7. a) If  $F$  is a field, then show that  $F[x]$  is a Euclidean domain. (8)
- b) Let  $R$  be a ufd. Then the product of two primitive polynomials over  $R$  is also a primitive polynomial. (8)
8. a) State and prove Gauss theorem. (12)
- b) Define  $R$  - module submodule and isomorphism of  $R$  - module. (4)
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Roll No. \_\_\_\_\_

[Total No. of Pages : 2

**PGIS-N-232 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Ordinary Differential Equations**  
**Paper : HCT - 1.3**  
**(New)**

Time : 3 Hours

Maximum Marks : 80

**Instructions to Candidates:**

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a) State and prove the existence theorem for the solution of second order initial value problem. (8)
- b) If  $\phi_1, \phi_2$  are two solutions of  $y'' + a_1(x)y' + a_2(x)y = 0$  in an interval I containing a point  $x_0$  then prove that  $W(\phi_1, \phi_2)(x) = e^{-\alpha_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$ . (8)
2. a) Let  $\phi_1, \phi_2$  be any two linearly independent solutions of  $L(y) = 0$  on an interval I, then show that every solution  $\phi$  of  $L(y) = 0$  can be written uniquely as  $\phi = C_1\phi_1 + C_2\phi_2$ , where  $C_1, C_2$  are constants. (8)
- b) Prove that two solutions  $\phi_1, \phi_2$  of  $L(y) = 0$  are linearly independent on an interval I if and only if  $W(\phi_1, \phi_2)(x) \neq 0$  for all  $x$  in I. (8)
3. a) Define adjoint and self adjoint equation. Further, transform  $(t^4 + t^2) \frac{d^2x}{dt^2} + 2t^3 \frac{dx}{dt} + 3x = 0$  into equivalent self adjoint equation. (8)
- b) Every solution  $\psi$  of  $L(y) = y'' + a_1y' + a_2y = b(x)$  on I can be written as  $\psi = \psi_p + C_1\phi_1 + C_2\phi_2$  where  $\psi_p$  is a particular solution,  $\phi_1, \phi_2$  are two solutions of  $L(y) = 0$  and  $C_1, C_2$  are constants. (8)



4. a) State and prove Sturm's separation theorem. (8)
- b) Show that the boundary value problem  $\frac{d^2y}{dx^2} + \lambda y = 0$  where  $y(0)=0$ ,  $y(\pi)=0$  is a Sturm-Liouville problem. (8)
5. a) Find the ordinary and singular point of the differential equation  $(x-1)y'' + xy' + \frac{1}{x}y = 0$ . (5)
- b) Show that  $x = 0$  and  $x = -1$  are singular points of the differential equation  $x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$ . (5)
- c) Define the following.
- Analytic function
  - Ordinary point of  $y'' + P(x)y' + Q(x)y = 0$ .
  - Singular point of  $y'' + P(x)y' + Q(x)y = 0$ . (6)
6. a) Explain power series solution about an ordinary point. (8)
- b) Use the power series method to find the general solution of  $(1-x^2)y'' + 2y = 0$ , with  $y(0) = 4$ ,  $y'(0) = 5$ . (8)
7. a) Explain Gram-Schmidt process of orthonormalization. (8)
- b) Define characteristic values and characteristic functions. Further, find characteristic values and characteristic functions of
- $$\frac{d}{dx} \left[ x \frac{dy}{dx} \right] + \frac{\lambda}{x} y = 0 \text{ with } y'(1) = 0, y'(e^{2\pi}) = 0, \lambda \geq 0. \quad (8)$$
8. a) Prove that Eigen functions corresponding to different eigen-values are orthogonal with respect to some weight function. (10)
- b) Define the following.
- Orthogonal set of functions.
  - Orthonormal set of functions. (6)

**PGIS-N-233 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**Discrete Mathematics**  
**Paper : HCT - 1.4**  
**(New)**

**Time : 3 Hours**

**Maximum Marks : 80**

**Instructions to Candidates:**

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a) What are logical connectives? Discuss the construction of compound statements with suitable examples (5)  
b) Prove that, there is no rational number  $p/q$  whose square is 2. (5)  
c) Show by mathematical induction that any finite nonempty set is countable. (6)
2. a) Determine the number of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and the remaining once white. (5)  
b) State and prove the principle of inclusion and exclusion for two sets. (6)  
c) Find the probability of occurrence of an odd number in an experiment of rolling a die. (5)
3. a) Define a numeric function and discuss the manipulation of numeric function with suitable examples. (5)  
b) Solve the recurrence relation  $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$ . (5)  
c) Solve the recurrence relation  $a_r - 4a_{r-1} + 4a_{r-2} = 0$  with  $a_0 = 1$  and  $a_1 = 6$ . (6)
4. a) Solve the recurrence relation  $a_r - 7a_{r-1} + 10a_{r-2} = 3^r$  with  $a_0 = 0$  and  $a_1 = 1$ . (8)  
b) Solve the recurrence relation  $a_r = 3a_{r-1} + 2, r \geq 1$  with the boundary condition  $a_0 = 1$  by the method of generating function. (8)

5. a) Define a binary relation and discuss its properties with suitable example. (8)
- b) If  $L$  be a lattice, then prove the following :
- $a \vee (a \wedge b) = a$
  - $a \wedge (a \vee b) = a$  (8)
6. a) Let  $L$  be bounded distributive lattice prove that, if a complement exists, it is unique. (5)
- b) If  $(L, \leq)$  is a Boolean algebra, then for all  $a, b \in L$ , prove that  $(a')' = a$ . (5)
- c) Let  $(A, \vee, \wedge, -)$  be a finite Boolean algebra. Let  $b$  be any non zero element in  $A$ , and  $a_1, a_2, \dots, a_k$  be all the atoms of  $A$  such that  $a_i \leq b$ . Then prove that  $b = a_1 \vee a_2 \vee \dots \vee a_k$ . (6)
7. a) Define the following and write an example of each
- monoid
  - group
  - abelian group
  - order of group. (8)
- b) State and prove Lagrange's theorem. (8)
8. a) Write a detailed note on error detection and error correction in the transmission of binary information. (6)
- b) Prove that an encoding function  $e : B^m \rightarrow B^n$  can detect  $K$  or fewer errors if and only if its minimum distance is at least  $(k+1)$ . (5)
- c) Show that, the encoding function  $e : B^3 \rightarrow B^6$  defined below is group code. (5)
- $e(000) = 000000$   
 $e(001) = 001100$   
 $e(010) = 010011$   
 $e(011) = 011111$   
 $e(100) = 100101$   
 $e(101) = 101001$   
 $e(110) = 110110$   
 $e(111) = 111010$

Roll No. \_\_\_\_\_

[Total No. of Pages : 2

**PGIS-N-234 A-21**  
**M.Sc. I Semester (CBCS) Degree Examination**  
**MATHEMATICS**  
**General Topology**  
**Paper : HCT - 1.5**  
**(New)**

**Time : 3 Hours**

**Maximum Marks : 80**

**Instructions to Candidates:**

1. Answer any **Five** full questions.
2. All questions carry **equal** marks.

1. a) Define closure of a set. Let A and B be subsets of a space X then prove The followings:
  - i.  $\bar{A}$  is the smallest closed set containing A.
  - ii. A is closed iff  $A = \bar{A}$
  - iii. If  $A \subset B$  then  $\bar{A} \subset \bar{B}$
  - iv.  $\overline{A \cup B} = \bar{A} \cup \bar{B}$  (8)
  - v.  $\overline{(\bar{A})} = \bar{A}$
- b) Define Interior of a set. Show that  $X - \bar{A} = (X - A)^o$ . (8)
2. a) Define Limit point of a set. If A & B are subsets of a space X, then prove the followings:
  - i. If  $A \subset B$  then  $D(A) \subset D(B)$
  - ii.  $D(A \cup B) = D(A) \cup D(B)$ . (8)
- b) Define a base. Let  $\beta$  be a collection of subsets of a non - empty set X. then prove that there exists a topology on X for which  $\beta$  is a base iff
  - i.  $X = \cup \{B : B \in \beta\}$
  - ii. If  $U, V \in \beta$  and  $P \in U \cap V$  then there is  $W \in \beta$  such that  $P \in W \subset U \cap V$ . (8)



3. a) Define a continuous mapping. Let  $X$  be any space Let  $T_1$  and  $T_2$  be two topologies on  $X$  then prove that the bijective map  $i: (X, T_1) \rightarrow (X, T_2)$  is continuous iff  $T_2 \subset T_1$ . (8)
- b) If  $f: X \rightarrow Y$  be a Continuous function and  $A \subset X$  then show that  $f|_A: A \rightarrow Y$  is continuous. Is the converse true? Justify. (8)
4. a) Define a  $T_2$  - space. If a topological space  $X$  is  $T_2$  and  $f: X \rightarrow Y$  is a closed bijection then show that  $Y$  is also a  $T_2$  - space. (8)
- b) Prove the following properties of a regular space are equivalent :
- $X$  is regular
  - For each  $P \in X$  and an open set  $U$  containing  $P$ , there is an open Set  $V$  such that  $P \in \bar{V} \subset U$ .
  - For each  $P \in X$  and a closed set  $F$  not containing  $P$ , there is an open set  $V$  such that  $P \in V$  and  $\bar{V} \cap F = \phi$ . (8)
5. a) Prove that  $X \times Y$  is regular iff  $X$  &  $Y$  are both regular spaces. (8)
- b) Define a normal space. Show that normality is a topological property. (8)
6. a) Define subsequence of a Sequence. Let  $f: N \rightarrow X$  converges to 'a' in  $X$  then prove that every subsequence of  $f$  in  $X$  converges to 'a'. (8)
- b) Prove that any continuous image of a connected space is Connected. (8)
7. a) Prove that every closed and bounded interval on  $\mathbb{R}$  is compact where  $\mathbb{R}$  has usual topology  $\mu$ . (8)
- b) Prove that the space  $X$  is compact iff every collection of closed sets with FIP has a non empty intersection. (8)
8. a) Define a Metric space. Prove that every separable metric space  $(X, d)$  is  $2^\circ$  - countable. (8)
- b) Define a Lindelof space. Prove that a metric space is Lindelof iff it is second countable. (8)