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PGIS-N 1018 B-15

M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Real Analysis)

Paper : HCT 1.1

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) Answer any Five questions.
2) All questions carry equal marks.

1. a) Define Riemann - steiltje's integrals and describe their existance. Prove that f is integrable with respect to α over [a,b] iff for every $\varepsilon > 0$ and for every partition P of [a,b] such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$

b) If $f \in \mathbb{R}(\alpha_1)$ and $f \in \mathbb{R}(\alpha_2)$ then show that $f \in \mathbb{R}(\alpha_1 + \alpha_2)$ and $\int_a^b f\,d(\alpha_1 + \alpha_2) = \int_a^b f d\alpha_1 + \int_a^b f d\alpha_2$ and if $f \in \mathbb{R}(\alpha)$ and 'c' is a positive constant then prove that $f \in \mathbb{R}(c\alpha)$ and $\int_a^b fd(c\alpha) = c\int_a^b fd\alpha$ **(8+8)**

2. a) If f is monotonic on [a,b] and α is continuous on [a,b] then show that $f \in \mathbb{R}(\alpha)$.

b) State and prove the first mean value theorem. **(8+8)**

3. a) Define the meaning of functions of bounded variation. Show that a bounded monotonic function is a function of bounded variation.

b) State and prove the fundamental theorem of calculus. **(8+8)**

4. a) If $\{f_n\}$ be a sequence of functions such that $\lim_{n\to\infty} f_n(x) = f(x), x \in [a,b]$ and let $M_n = \sup|f_n(x) - f(x)|, x \in [a,b]$ then prove that $f_n \to f$ uniformly on [a,b] if and only if $M_n \to 0$ as $n \to \infty$.

b) If a series $\sum_{n=}^{\infty} f_n$ converges uniformly to f in [a,b] and x_0 is a point in [a,b] such that $\lim_{x \to x_0} f_n(x) = a_n, n = 1,2,3,......$ then prove the followings:

i) $\sum_{n=1}^{\infty} a_n$ Converges.

ii) $\lim_{x \to x_0} f(x) = \sum_{n=1}^{\infty} a_n$ **(8+8)**

5. a) If a series Σf_n converges uniformly to f in an interval [a,b] and its terms f_n are continuous at a point x_0 of the interval then prove that the sum function f is also continuous at x_0.

b) If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,.....$, and if $\{f_n\}$ Converges uniformly on K then show that $\{f_n\}$ is equi continuous on k. **(8+8)**

6. State and prove the stone - weierstrass theorem. **(16)**

7. a) Prove the followings:

i) If $T \in L(R^n, R^m)$, then $\|T\| < \infty$ and T is a uniformly continuous mapping of R^n into R^m.

ii) If $T, S \in L(R^n, R^m)$ and C is a scalar,

then $\|T+S\| \leq \|T\| + \|S\|$

$\|CT\| = |C| \ \|T\|$

iii) If $T \in L(R^n, R^m)$ and $S \in L(R^m, R^k)$ then $\|ST\| \leq \|S\| \ \|T\|$

b) If f is a function of class $C^{(1)}$ then prove that f is a differentiable function. **(8+8)**

8. State and prove the implicit function theorem. **(16)**

PGIS-N 1020 B-15

M.A/M.Sc Ist Semester (CBCS) Degree Examination

Mathematics

(Algebra - I)

Paper - HCT 1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

1. Answer any **five** full questions
2. All questions carry **equal** marks.

1. a) Prove that any subgroup of an infinite cyclic group is also infinite cyclic. Also prove that an infinite cyclic. Group has exactly two generators **(8)**

b) Show that every permutation $\sigma \in S_n$, where S_n is the symmetric group and it can be expressed as a product of disjoint cycles. **(8)**

2. a) State and prove Cayley's theorem **(8)**

b) Derive the class equation for finite group. **(8)**

3. a) Prove that a group G is solvable if and only if $\overset{(n)}{G} = \{e\}$ for some $n \geq 1$ **(8)**

b) Show that a group of prime order is solvable. Also prove for a group G with K normal subgroup such that both K and G/K are solvable, then G is solvable. **(8)**

4. Prove that every integrable domain can be embedded in a field. **(16)**

5. a) Let R be an Euclidean domain then show that for any $a \in R$ Which is not a unit can be expressed as a product of irreducible elements **(8)**

b) If R is commutative ring with unit element, then show that R $[x]$ is also commutative ring. If R is an integral domain then prove that R$[x]$ is also an integral domain. **(8)**

6. a) Show that if F is a field, then F $[x]$ is a Euclidean domain (8)

b) Let R be a unique factorisation domain. Then prove that the polynomial ring R $[x]$, is a unique factorisation (8)

7. a) Prove that $F(\alpha)$ has dimension n as a vector space over F. (8)

b) Let K/F and L/K be algebraic extension then prove that L/F is an algebraic extension (8)

8. a) Let $f(x) \in F[x]$ be of degree n. Then show that $f(x)$ has a splitting field. (8)

b) For a finite field F with P^n elements. Prove that F has a subfield F' with P^m elements if and only if m divides n. (8)

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PGIS-N 1029 B-15

M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(General Topology)

Paper - HCT - 1.5

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any **five** full questions.
2) All questions carry equal marks.

1. a) Define a topology τ on a non-empty set X . Let μ consist of ϕ and all those subset G of a real line $\mathbb{R}$ such that to each $x \in G$ there exists $\in > 0$ such that $(x-\in, x+\in) \subset G$. Then show that μ is a topology on $\mathbb{R}$ **(8)**

b) using the Kuratowski's closure axioms in a topological space X, then prove that there exists a topology τ on x such that $C(A) = \overline{A}$ **(8)**

2. a) Define neighbourhood of a point. Prove that a subset G of a space X is open iff it is a neighborhood of its points. **(8)**

b) If A is a subset of a space X then prove the followings: **(8)**

i) $A \cup D(A)$ is closed.

ii) $\overline{A} = A \cup D(A)$

3. a) Define base for a topology. Then prove the following two properties on a base β are equivalent: **(8)**

i) β is a base for τ .

ii) For each $G \in \tau$ and $P \in G$ there is $U \in \beta$ such that $P \in U \subset G$

b) Let X, Y be the topological spaces and $f : X \to Y$ be a mapping then show that f is closed iff $\overline{f(A)} \subset f(\overline{A})$ *for* $A \subset X$. **(8)**

4. a) Define a T_2- space. If a topological space X is T_2 and $f: X \to Y$ is a closed bijection then show that y is also a T_2-space. (8)

b) Prove the following properties of a regular space are equivalent:

i) X is regular

ii) For each $P \in X$ and an open set U containing P, there is an open set V such that $P \in \bar{V} \subset U$

iii) For each $P \in X$ and a closed set F not containing P, there is an open set V such that $P \in V$ and $\bar{V} \cap F = \phi$. (8)

5. a) Define a normal space. Show that normality is a topological property. (8)

b) Define separable space. Show that every 2^0- countable space is separable. (8)

6. a) Define a connected space. Prove that the following conditions are equivalent:

i) The space X is connected.

ii) The only subsets of X which are both open and closed are ϕ & X

iii) No continuous mapping $f: X \to \{0,1\}$ is subjective. (8)

b) If $\{A_\alpha : \alpha \in D\}$ be a family of connected subsets of a space X such that one of the members of this family intersects every other member then prove that $U\{A_\alpha : \alpha \in D\}$ is connected. (8)

7. a) Define a compact space. If A be a compact subset of a hausdorff space X and $P \notin A$ prove that there exists disjoined open sets U & V such that $P \in V$ and $A \subset U$. (8)

b) Prove that the space X is compact if and only if every collection of closed sets with finite intersection property has a non empty intersection. (8)

8. a) Define a metric space. Show that in any metric space the set of all open spheres is a base for topology on X. (8)

b) Prove that a metric space is Lindelof if and only if it is 2^0 countable. (8)

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PGIS - N 1026 B - 15
M.A/M.Sc Ist Semester (CBCS) Degree Examination
Mathematics
(Classical Mechanics)
Paper - SCT 1.1
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

1. Answer any **five** full questions
2. All questions carry **equal** marks.

1. a) What are different kinds of constraints of dynamical system explain . (8)

b) State and prove D'Alembert's principle. (8)

2. a) Deduce Lagronge's equation for impulsive motion. (8)

b) Derive energy equation for impulsive motion. (8)

3. a) Construct Lagragian and hence equation of motion of a simple pendulum placed in a uniform gravitational field. **(8)**

b) Define rigid body. Find the momentum inertia and product inertia of a body at O. **(8)**

4. a) Explain transformations associated with Eulerian angles. **(8)**

b) A symmetrical top can turn freely about a fixed point in its axis of symmetry and is acted on by forces derived from the potential function $\mu \cot^2 \theta, \theta$ is the angle between this axis and a fixed line, say O Z. Show that the equation of motion can be integrated in terms of elementary functions. **(8)**

5. a) Derive hamilton's canonical equation from hamilton's principle (8)

b) State lee How - Chung theorem and discuss poincare integral invariant (8)

6. a) Obtain Hamilton - Jacobi equations for simple harmonic motion and find a complete integral and determine solution of it. (8)

b) Prove that F,G are both integrals of motion, them so is their POISSON bracket. (8)

7. a) A particle of mass m is falling under gravity. Solve for the motion of the particle using canonical transformation (8)

b) Prove that lagrange's bracket is invariant under canonical transformation. (8)

8. a) Show that Lagrange's bracket donot obey the commutative law. Also prove the fundamental Lagranges bracket. (8)

b) Find the relationship between lagrange's and poissons bracket (8)

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PGIS-N 1028 B-15

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Fuzzy Sets and Fuzzy Systems)

Paper - SCT - 1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates.

1) Answer any five questions
2) All questions carry equal marks.

1. a) State and prove the De-Morgans laws for crisp sets (6)

b) Define a fuzzy set and explain with suitable examples (5)

c) What is the support of a fuzzy set? Explain with example (5)

2. a) Following are the fuzzy sets defined on the set X={5,10,20,30,40,50,60,70,80} of ages

$$\text{Adult} = \frac{0.8}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} + \frac{1}{60} + \frac{1}{60} + \frac{1}{80}$$

$$\text{Young} = \frac{1}{5} + \frac{1}{10} + \frac{0.8}{20} + \frac{0.5}{30} + \frac{0.2}{40} + \frac{0.1}{50}$$

$\text{Old} = \frac{0.1}{20} + \frac{0.2}{30} + \frac{0.4}{40} + \frac{0.6}{50} + \frac{0.8}{60} + \frac{1}{70} + \frac{1}{80}$. Find compliment of 'Adult' and verify that De-Morgans laws satisfied for the sets 'Young' and 'Old' (8)

b) Prove that a fuzzy set A on $\mathbb{R}$ is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min(A(x_1), A(x_2))$ for all $x_1, x_2 \in \mathbb{R}$ *and* $\lambda \in [0,1]$ (8)

3. a) If $A, B \in F(X)$ then prove that the following properties hold for all $\alpha \in [0,1]$

i) ${}^{\alpha+}(A \cap B) = {}^{\alpha+}A \cap {}^{\alpha+}B$

ii) ${}^{\alpha+}(A \cup B) = {}^{\alpha+}A \cup {}^{\alpha+}B$

b) If A, B are the two fuzzy sets defined on the universal set X then prove that the following properties hold for all $\alpha \in [0,1]$

i) A=B iff $\alpha_A = \alpha_B$

ii) A=B iff ${}^{\alpha-}A = {}^{\alpha-}B$

4. a) State and prove first decomposition theorem (8)

b) Write the axiomatic definition of fuzzy complement and show that the fuzzy complement defined by $C_\lambda(a) = \frac{1-a}{1+\lambda a}$ for all $a \in [0,1]$ and $\lambda \in (-1,\infty)$ is a involutive fuzzy complement. (8)

5. a) Prove that every fuzzy complement has at most one equilibrium (8)

b) Define Archimedian t-norm and show that the standard fuzzy intersection is the only idempotent t-norm (8)

6. a) If i_w denote the class of yager t-norms defined by $i_w(a,b) = 1 - \min\left(1, \left[(1-a)^w + (1-b)^w\right]^{\frac{1}{w}}\right)$, w>0 then prove that

$i_{min}(a,b) \leq i_w(a,b) \leq \min(a,b)$ for all $a,b \in [0,1]$ (8)

b) Define Archimedian t-conorm and prove that the yager class of t-conorms covers the whole range of t-conorms (8)

7. a) Given a t-norm i and an involutive fuzzy complement C, then prove that the binary operation u on [0,1] defined by $u(a,b) = c(i(c(a),c(b)))$ for all $a,b \in [0,1]$ is a t-conorm such that <i,u,c> is a dual triple (8)

b) Define a fuzzy number and explain the concept of "large number" and "small number" with suitable example (8)

8. Write notes on the following

i) Linguistic variables

ii) Arithmetic operation on fuzzy numbers

iii) Fuzzy equations (6+5+5)