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PGIVS 1505 A-18
M.A./M.Sc. IVth Semester Examination
MATHEMATICS
(Differential Geometry) (CBCS)
Paper : HCT-4.4

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Answer any **FIVE** questions.
2. All questions carry equal marks.

1. a) If f and g are the functions on E^3, v_p & w_p, are the tangent vectors, a,b are the numbers then prove the following:

i) $(av_p + bw_p)[f] = av_p[f] + bw_p[f]$

ii) $v_p[af + bg] = av_p[f] + bv_p[g]$

iii) $v_p[fg] = v_p[f].g(p) + f(p).v_p[g]$. **(8)**

b) Define a curve in E^3. Show that the curves given by $(t, 1+t^2, t)$ and $(\sin t, \cos t, t)$ have the same initial velocity v_p. If $f = x^2 - y^2 + z^2$ then compute $v_p[f]$ by evaluating f on each of these curves **(8)**

2. a) For any three 1-forms $\phi_i = \sum_j f_{ij} d_{xj}$ $(1 \le i \le 3)$ then prove that

$$\phi_1 \wedge \phi_2 \wedge \phi_3 = \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} dx_1\, dx_2\, dx_3$$ **(8)**

b) Let $F = (f_1, f_2, ..., f_m)$ be a mapping from E^n to E^m. If v is a tangent vector to E^n at p then prove that $F_*(v) = (v[f_1], ..., v[f_m])$ at $F(P)$. **(8)**

3. a) If (e_1, e_2, e_3) be a frame at a point p of E^3. If v is any tangent vector to E^3 at p then derive the orthogonal expansion about v as

$v = (v.e_1)e_1 + (v.e_2)e_2 + (v.e_3)e_3$. **(8)**

b) If β is an unit speed curve with constant curvature $k > 0$ and torsion zero then show that β is a part of a circle of radius $\frac{1}{k}$ units. **(8)**

4. a) Test for a curve β to be a spherical curve has curvature $k \geq \frac{1}{a}$, where 'a' is the radius of the sphere. **(8)**

b) Define a cylindrical helix. Prove that a regular curve α with $k > 0$ is a cylindrical helix if and only if $\frac{\tau}{k}$ is constant. **(8)**

5. a) Define an isometry and orthogonal transformation of E^3. If $C: E^3 \to E^3$ is an orthogonal transformation then show that C is an isometry of E^3 **(8)**

b) If F is an isometry of E^3 then prove that there exists a unique translation T and α unique orthogonal transformation C such that $F = TC$ **(8)**

6. a) Let F be an isometry of E^3, with orthogonal part C. Then prove that $F_*(v_p) = (cv)_{f(p)}$ for all tangent vectors v_p to E^3 **(6)**

b) If $\alpha, \beta : I \to E^3$ are unit speed curves such that $k_\alpha = k_\beta$ and $\tau_\alpha = \pm\tau_\beta$. then prove that α & β are congruent curves. **(10)**

7. a) Prove that every sphere in E^3 is a surface in E^3. **(8)**

b) Let f be a real valued differentiable function on a non empty open set D of E^2 then prove that the function $X : D \to E^3$ satisfying $X(u,v) = (u, v, f(u,v))$ is a proper patch. **(8)**

8. a) Explain the stereo graphic projection of the punctured sphere S onto the plane. **(8)**

b) Prove that a mapping $X : D \to E^3$ is regular iff $X_u(d)$ & $X_v(d)$ are the u,v partial derivatives of $X(u,v) = X(d)$ are linearly independent $\forall d \in D$, where $D \subset E^2$. **(8)**

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PGIVS-1508-A-18

M.A./M.Sc. IVth Semester Examination

MATHEMATICS

(Computational Numerical Methods-II) (CBCS)

Paper : HCT-4.3

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. *Answer any **FIVE** questions.*
2. *All questions carry **equal** marks.*

1. a) Solve by using Runge-Kutta method of fourth order **(8)**

$y'' = xy' - y$; $y(0) = 3$ *and* $y'(0) = 0$ and compute the approximate value of $y(0.1)$.

b) Write a detailed note on step-size control of Runge-Kutta fourth order method. **(8)**

2. a) Given $y' = x^2(1+y)$ *and* $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by using Milne's predictor corrector method. **(8)**

b) Given $y' = y - x^2$, $y(0) = 1$, $y(0.2) = 1.1218$, $y(0.4) = 1.4682$, $y(0.6) = 1.7379$, find $y(0.8)$ by using Adam-Bashforth predictor - corrector method. **(8)**

3. a) Write the classification of general second order PDE and discuss the classification of physical problems. **(8)**

b) Describe the explicit finite difference scheme for solving parabolic partial differential equation. **(8)**

4. a) Use the crank-Nicolson method (Gauss-Seidal iteration) to solve $u_t = u_{xx}$ with the initial and boundary conditions $u(0,t) = 0; u(1,t) = t$, $u(x,0) = 0$ for $0 \le x \le 1$ and take $h = 0.2$ *and* $k = 0.04$ **(10)**

b) Discuss the successive over Relaxation scheme for the crank-Nicolson method to solve parabolic partial differential equation. **(6)**

5. a) Describe alternating direction explicit method to solve $u_t = K(u_{xx} + u_{yy})$ in a rectangular region $0 \le x \le a, 0 \le y \le b$. With given initial and boundary conditions. **(8)**

b) Write a note on parabolic equation in cylindrical and spherical co-ordinates. **(8)**

6. a) Derive a standard diagonal five point formula to solve the Laplace equation $u_{xx} + u_{yy} = 0$. **(6)**

b) Solve $u_{xx} + u_{yy} = 0$. with the boundary conditions $u(0,y) = 0; u(x,0) = 0$, $u(x,1) = 100x; u(1,y) = 100y$. with a square grid size h=1/3. **(10)**

7. Solve the Poissons equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = gu;\ \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}$ subject to boundary conditions

$$u = x \quad at \quad y = 0$$
$$u = x + 1 \quad at \quad y = 1$$

$$\frac{\partial u}{\partial x} = -2u - y \quad \left.\begin{matrix} at\ x = 0 \\ at\ x = 1 \end{matrix}\right\} 0 < y < 1.$$ **(16)**

8. a) Solve $u_{tt} = u_{xx}$ for $t > 0,\ 0 < x < 1$ by finite difference method with initial conditions $(t = 0)$

$$u = 5(x - 0.3), 0.3 < x < 0.5$$
$$= 5(0.7 - x); 0.5 < x < 0.7$$
$$= 0 \text{ for all other points}$$

$\frac{\partial u}{\partial t} = 0$ at $t = 0$ and boundary conditions. $u(0,t) = 0$ *and* $u(1,t) = 0$ **(10)**

b) Write a note on implicit finite difference method to solve hyperbolic partial differential equation. **(6)**

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PGIVS 1507 A-18

M.A./M.Sc. IVth Semester (CBCS) Degree Examination

Mathematics

(Graph Theory-II)

Paper : HCT-4.2

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. Answer any **five** full questions.
2. All questions carry equal marks.

1. a) Define join and product operations. Draw Join, product and composition of the following graphs.

i) $\overline{C}_4 + \overline{K}_3$ ii) $K_{1,2} + \overline{P}_2$ iii) $K_2 \times K_2$

iv) $K_3 \times K_3$ v) $P_2[P_3]$ vi) $K_{1,2}[P_2]$

Show that if G is a maximal planar (p,q) graph with $p \geq 3$, then $q = 3p - 6$. **(8)**

b) Prove that a graph is outerplanar if and only if it has no subgraph homeomorphic to K_4 *or* $K_{2,3}$. **(8)**

2. a) Define underlying graph. If G is a (p,q) graph which is plane graph in which every face is n-cycle, then prove that $q = \dfrac{n(p-2)}{n-2}$. **(8)**

b) Prove that $Cr(K_{2,2,3}) = 2$ and determine $Cr(K_{2,2,2})$ **(8)**

3. a) Show that every tree with two or more vertices is bicolorable. Find the chromatic number of

i) $\overline{K}_p$ ii) P_5 iii) C_{16} iv) C_{13} **(8)**

b) Prove that for any graph G, the sum and product of χ and $\overline{\chi}$ satisfy the inequalities.

$2\sqrt{p} \leq \chi + \overline{\chi} \leq p = 1,\ p \leq \chi\overline{\chi} \leq \left(\dfrac{p+1}{2}\right)^2$ **(8)**

4. a) Write the simple sequential algorithm. Find the chromatic number of cubic graph with $p = 8,10$ vertices which does not contains triangle and contains triangle. **(8)**

b) Show that for any graph G, $\chi(G) \le \Delta(G) + 1$. **(8)**

5. a) Prove that a map G is K-face colorable if and only if its dual G^{*} is K-vertex colorable. **(8)**

b) Show that a map is 2-face colorable if and only if it is an eulerian graph. **(8)**

6. a) For any nontrivial connected graph G, prove that $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$ **(8)**

b) Show that for every positive integer n, the graph K_{2n+1} can be factored into n-hamilton cycles. **(8)**

7. a) Prove that the independent set, clique and vertex cover are NP-complete. **(8)**

b) Show that a dominating set S is a minimal dominating set if and only if each vertex $u \in S$. One of the following conditions holds.

i) u is an isolated of S

ii) There exists a vertex $v \in V - S$ for which $N(v) \cap S = \{u\}$ **(8)**

8. a) Prove that for any graph G, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \le \surd(G) \le n - \Delta(G)$ **(8)**

b) For any tree T, show that $\gamma(T) = n - \Delta(T)$ if and only if T is a wounded spider. **(8)**

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PGIVS 1509-A-18
M.A./M.Sc. IVth Semester Examination
MATHEMATICS
(Fluid Mechanics-II)
Paper : SCT-4.1
(CBCS)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. Answer any **FIVE** questions.
2. All questions carry equal marks.

1. a) Explain Continuum hypothesis. Derive mass conservation equation. **(8)**

b) Prove that in a "Hydrostatic Stress System" the fluid pressure is equal to the arithmetic mean of the normal stresses taken with negative sign. **(8)**

2. a) Derive the following relation $\tau_{ij} = \left(-p + \lambda \frac{\partial v_k}{\partial x_k}\right)\delta_{ij} + \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ **(8)**

b) In certain flow obeying stokes law, the velocity field is given by $v_1 = 4x_1 x_2 x_2, v_2 = x_3^2, v_3 = -2x_2 x_3^2$. Find the strain rate tensor and stress tensor. **(8)**

3. Derive Energy equation. **(16)**

4. a) Derive vorticity equation **(8)**

b) State and prove principle of similarity. **(8)**

5. a) Define Reynolds number, froude number, Mach number, prandtl number, peclet number and grashof number. **(8)**

b) Explain the steady flow in a straight conduit. **(8)**

6. a) State and prove stokes first problem. **(8)**

b) Prove that for steady two dimensional creeping flow, the stream function is biharmonic. **(8)**

7. a) Derive $2f''' + ff'' = 0$. **(8)**

b) Derive boundary layer equations. **(8)**

8. a) Define boundary layer thickness, displacement thickness and momentum thickness. **(8)**

b) Derive von-karman's momentum integral equation. **(8)**

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PGIVS 1506 A-18
M.A./M.Sc. IVth Semester Examination
MATHEMATICS
(Measure Theory) (CBCS)
Paper : HCT 4.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1. *Answer any **Five** questions.*
2. *All questions carry **Equal** marks.*

1. a) Define outer measure. Let X be a space of atleast two points and $x_0 \in X$. For each $A \subset X$, defined $\mu(A) = \begin{cases} 0 & if \quad x_0 \notin A \\ 1 & if \quad x_0 \in A \end{cases}$ then prove that μ is an outer measure. **(8)**

b) Show that if the outer measure of a set is zero, then the set is measurable. **(8)**

2. a) Define Boolean ring and algebra. Show that a Boolean ring B containing X is an algebra. Also show that every algebra is a ring. **(8)**

b) Define exterior and interior measure of a set. Show that $m_e(A) \geq m_i(A)$ for any set A. **(8)**

3. a) If E_1 *and* E_2 are measurable then prove that $E_1 \cup E_2$ IS measurable. What do say about $E_1 \cap E_2$ is measurable? Justify. **(8)**

b) State and prove the second fundamental theorem. **(8)**

4. a) Define a measurable function. If f is a constant function over a measurable set E then show that f is measurable over E. **(8)**

b) Let f and g be measurable functions defined over a measurable set E. show that $f+g$, $f-g$, fg and $\frac{f}{g}$ (if g vanishes no where on E) are measurable functions. **(8)**

5. a) State and prove the Lebesgue bounded convergence theorem. **(10)**

b) Using the Lebesgue's dominated convergence theorem, evaluate the following integral,

$\lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ where $f_n(x) = \frac{n^{3/2}x}{1+n^2x^2}$, $0 \le x \le 1$ for $n = 1, 2, 3....$ **(6)**

6. a) Define a differentiable function. If $f(x)$ is continuous and integrable function then prove that $F'(x) = f(x)$ a.e. where $F(x)$ is a differentiable function. **(8)**

b) Define absolutely continuous function. Prove that an indefinite integral is an absolutely continuous function. **(8)**

7. a) Show that every absolutely continuous function is an indefinite integral of its own derivative. **(8)**

b) If f and g are integrable functions over $[a,b]$. Suppose F and G are indefinite integrals then prove that $\int_a^b F(t)g(t)\,dt + \int_a^b f(t)\,G(t)\,dt = F(b)G(b) - F(a)G(a)$ where $t \in [a,b]$ **(8)**

8. a) Define a positive set. Show that a union of any countable collection of positive subsets of X is positive. **(6)**

b) State and prove Hahn Decomposition theorem. **(10)**
